

Improving pelican optimization algorithm for solving integer and mixed integer optimization problems

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Abstract Mixed-integer linear programming (MILP) problems, prevalent in logistics, scheduling, and resource allocation, pose significant computational challenges due to their NP-hard nature, prompting the need for efficient metaheuristic approaches. This study proposes an enhanced pelican optimization algorithm (POA) that adapts the original population-based method inspired by pelican hunting strategies of prey approach (exploration) and surface winging (exploitation) for discrete domains via novel tent-shaped transfer functions. These functions enable seamless discretization of continuous solutions into binary and integer variables, offering computational simplicity and gradual curvature superior to traditional S- and V-shaped transfers. Tent-shaped functions outperformed others, yielding lowest standard deviation values and means closest to optima, demonstrating superior stability and precision for practical optimization.

Keywords linear programming, mixed integer problem, pelican optimization algorithm, transfer function

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1. Introduction

The optimization problems in the real world are usually very difficult to solve and most of the applications must contend with problems. Optimization tools must be applied to address such issues although it is not guaranteed that the optimal solution may be found [1]. Also, new algorithms have been designed to determine whether they can handle such difficult optimization problems. Some of these new algorithms include particle algorithms. Swarm optimization, cuckoo search and firefly algorithm have become popular because they are very high efficiency [2, 3, 4]. In this article, we have applied pelican algorithm the following general form can be expressed a problem as:

$$\text{Max } c^T x \quad (1)$$

$$\text{s.t. } Ax \leq b \quad (2)$$

$$x \in Z^n \quad (3)$$

where the solution $x \in Z^n$ is a vector of n integer variables: $x = (x_1, x_2, \dots, x_n)^T$ and the data are rational and are given by the $m \times n$ matrix A , the $1 \times n$ matrix c , and the $m \times 1$ matrix b . This formulation includes also equality constraint, because each equality constraint can be represented by means of two inequality constraints like those included in Eq. (2).

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A large number of real-life optimization problems (transport problems, problems of scheduling and distribution, resources planning, etc.) are offered by linear mixed integer models. The integer variables are specific items, which cannot be divided the continuous variables reflect (number of machines, vehicles and others).

Nowadays Mixed the application of Integer Problems (MIP) is decision oriented in most industrial and business operations. This is not only because of the increased both to the computing power of contemporary computers, and to better MIP solvers. facilitate simpler construction and solving of such models [5, 6, 7, 8].

The integer problems which have not continuous computing complexity the combination nature of the variables is the source of their combination nature. In mixed integer problems the computing complexity higher, as it is required to determine both the optimal sum of the integer variables and the optimum value of the continuous variables. The issues of this kind are part of the Non-deterministic Polynomial-time (NP) class. The famous ways of optimal solution finding (like branch and-bound or cutting planes) are exponentially complex in computing need a lot of computing time even on not large problems [9, 10]. These The existence of a large number of heuristics has been caused by computing problems algorithms. Heuristics play a very significant role in applied optimization [11, 12]. They can only be applied in the solution of large and complex problems, which are met very often practice. Their primary aim is to guarantee reasonable time of solutions of the optimization problem, near optimal feasible solution. In addition, they are able to be very helpful in complicated accurate procedures of determining possible initial solutions as well as to accelerate the search procedure at specified stages of such techniques (e.g. in a node of the branch and bound tree) [13, 14, 15]. The heuristic algorithms can be separated into two categories: problem-oriented, having a particular structure [16, 17] and general purpose techniques . To be applicable, it must be used in order to find application in the heuristic algorithms are tested against commercial software products [10]. their effectiveness, and to the universality of techniques, i.e., to their applicability and success on various types of problems [8, 11, 18].

2. Mixed Integer Programming

The term linear integer programming is used to refer to the family of combination constrained optimization problems whose variables are integer, whose objective is a linear function and whose constraints are linear inequalities. Linear Integer Programming (or LIP) optimization Integer programming is used to solve the problem posed by the non-integer solutions in the cases where integer values are needed. rounding might influence the profit or the cost by millions of dollars. Here we must have the problem solved in such a way that an optimal integer solution is ensured. Integer programming: as a pure integer linear programming, in which all the variables are integer-valued, it provides the possibility to find integer values. has to be integer, or as a mixed integer linear programming that permits some that the variables are continuous, or a 0-1 integer model, the decision variables are integer valued with zero or one. Many real-life problems in logistics, economics, social sciences and politics can be modeled as linear integer optimization problems. Other combination problems as the knapsack-capital budgeting problem, warehouse location problem, travelling salesman problem, decreasing costs and machinery selection problem, network and graph problems, such as maximum flow problems, set covering problems, matching problems, weighted matching problems, spanning tree problems and many scheduling problems can also be formulated as linear integer optimization problems. Cutting plane techniques are exact integer programming techniques [19, 20].

The branch and the bound are both computationally expensive, in large scale problems [19, 20]. The branch and the bound algorithms are very superior to the algorithms that utilize cutting planes alone. An example of such benefits is that the algorithms can be terminated prematurely provided at least one integral solution has been identified and a solution can be provided which is achievable even though it may not be optimal. In addition, the LP relaxations provide a worst-case estimate of the distance to optimal of the returned solution using their solutions. Lastly, multiple optimal solutions can be returned by the branch method and the bound method [2].

The integer linear programming is NP complete, hence due to this reason a large number of problems are intractable. Therefore, the heuristic methods have to be employed instead of the integer linear programming. As an example, Swarm intelligence meta heuristics, one of which is an ant colony optimization, artificial bee colony

optimization particle swarm optimization .To solve integer programming problems, most of the heuristics truncate or round the real valued solutions to the nearest integer values [2, 19, 20].

Mixed Integer Linear Programs (MILPs) are mathematical optimization problems that have two types of variables, discrete and continuous variables. The objective is also linear as well as the constraints. This class of optimization problems arises in numerous real-life problems across fields. In fact, MILP models can be used to formulate many real problems, such as: packing, knapsack, inventory, production planning, location, resource allocation, routing and scheduling problems, just to mention but a few [21]. Such a wide applicability has made the development of efficient algorithms to solve this general and popular type of optimization problems even more interesting. MILP problems are NP-hard. The impossibility to solve a great variety of important optimization problems has led to the development of approximation algorithms [22].

It is very common to be faced with the situation that the problem is NP-hard and it is really hard to find an optimal solution within a reasonable amount of time [23]. In general solving MILPS depend on two from of algorithms: exact algorithms and heuristics, to all intents and purposes. (e.g. branch and bound, branch and cut, branch and price) tend to be generally applicable, but have been found to be tedious on large or more complicated problems. Heuristics and especially meta heuristics can be employed when the instances are too large or challenging to solve precisely. A meta heuristic is an overarching process to choose between various heuristics. It is possible to consider two categories of meta heuristics: single solution algorithms (e.g. local search, tabu search) and population-based algorithms (e.g. evolutionary algorithms, swarm optimization) [22, 24, 25, 26].

We consider a generic mixed integer linear program (MILP) in the form

$$\min c^T x \quad (4)$$

$$Ax \geq b \quad (5)$$

$$x_j \in \{0, 1\} \quad \forall j \in B, \quad (6)$$

$$x_j \text{ integer } \forall j \in G \quad (7)$$

$$x_j \text{ continuous } \forall j \in C \quad (8)$$

where A is an $m \times n$ input matrix, b and c are input vectors of dimension m and n , respectively. Here, the variable index set $N := \{1, \dots, n\}$ is partitioned into (B, G, C) , where B is the index set of the 0 – 1 variables (if any), while the sets G and C index the general integer and the continuous variables, respectively. Bound on the variables, including the 0 – 1 bounds on the binary ones, are assumed to be part of system Eq. (5). Removing the integrity requirement on variables indexed by $I := B \cup G$ leads to the LP relaxation $\min \{c^T x : x \in P\}$ where $P = \{x \in \mathbb{R}^n : Ax \geq b\}$. MILP heuristics aim at finding a feasible (and hopefully good) solution of the problem above, which is an NP-hard problem by itself [22].

3. Literature Review

A large number of practical optimization problems have decision variables whose values must be integer. Such problems are numerous in applications; e.g. plant operation, design, location, scheduling and set covering, set partitioning, set packing problems and allocation models [27]. The previous approaches to IP problem solutions belong to [28] and to [29]. These algorithms are aimed at solving restricted sub classes of IP problems and not the general type of IPPs. The overall IP issues in certain cases may be, approximated and solved as a linear programming (LP) problem and the solution to the LP optimization problem rounded off. Such a strategy can offer a viable solution, yet the solution might not be close to the best one. To solve integer linear programming problems (ILP) many methods have been proposed such as branch and bound (B&B), cutting planes, polyhedral

developments, hybrid algorithms, reformulation- linearization technique (RLT), facial disjunctive programming and post-solution analysis [30, 46, 47, 48].

The optimization of reliability problems with separable objective function and numerous constraints have also done much work. [31] give such examples. The majority of them will not give a global optimum answer. An approach called 0-1 integer nonlinear programming [32] is the optimization technique developed to solve integer nonlinear programming problems (INLP) variables. Their approach is a near relation to the lexicographic approach to the knapsack problem by [33], and the additive algorithm, introduced by [34]. [32] presuppose the objective function to be a monotone non decreasing one and suppose that the functional constraints may be written (rewritten) in the form of a difference between two functions that happen to be monotone non decreasing. [35] came up with an algorithm to solve a spare allocation problem that can be used in the integer problem solved using a non-0-1 variable. In their approach, they termed it New Lawler and Bell. Generalization Using [32] algorithm, [36] generalized the Lawler and Bell algorithm, which solves a larger class of INLP problems. [37] provided two phase based optimization operations to integer programming problems. [38] introduced a survey on the non-standard methods of the integer programming problems.

A number of computational methods (including B&B, cutting planes, relaxation, outer approximation etc.) that are reasonably effective when used on many problems have been suggested in the literature on ILPs. Nevertheless, no computational algorithm has been developed so far that could say that it can effectively solve a large number of INLPs [39].

4. Pelican Optimization Algorithm (POA)

The enormous pelican uses a big bag in its throat to catch and consume prey. It also has a long beak. This bird, which inhabits groups of several hundred pelicans, enjoys social interactions and group living. Pelicans have the following physical characteristics: they are between 2.75 and 15 kg in weight, 1.06 and 1.83 m in height, and 0.5 and 3 m in wingspan [40, 41]. Fish is a pelican's primary dietary source, with frogs, turtles, and crabs appearing less frequently [42]. When extremely hungry, pelicans may even consume seafood. Pelicans frequently cooperate during hunting. Once they have located their meal, the pelicans dive from a height of 10 to 20 meters to reach it. Lower altitudes are where certain creatures hunt [43]. They then extend their wings over the water to deceive fish into areas of shallower water, which makes it easier for them to catch and consume the fish. The pelican must bend its head forward before swallowing to prevent water from getting into its beak as it catches fish. Pelicans' astute hunting behaviors and strategies have allowed them to develop into expert hunters. The main source of inspiration for the design of POA [44, 45]. of course, some species also fly lower to reach their food. The fish are then forced to move to shallow water by their spreading wings on the water's surface, which makes it easier for them to catch fish. When a pelican catches a fish, a lot of water gets into its beak; to get rid of the extra water, it tilts its head forward and swallows the fish [51, 49, 50]. The cognitive process of pelican behavior and hunting strategy has made these birds adept hunters. The previously indicated strategy's modeling served as the primary source of inspiration for the design of POA [41, 53, 54, 55].

Using a population-based approach, the proposed POA counts pelicans as members of its population. Every person in a population-based algorithm represents a potential solution. Each population member suggests parameter values for the optimization problem based on their search space position. We first use Eq. (9) to randomly initialize population members based on the problem's upper and lower bounds.

$$x_{i,j} = l_j + \text{rand.} (u_j - l_j), \quad i = 1, 2, \dots, N, \quad j = 1, 2, \dots, m \quad (9)$$

where $x_{i,j}$ is the value of the j th variable specified by the i th candidate solution, N is the number of population members, m is the number of problem variables, rand is a random number in interval $[0, 1]$, l_j is the j th lower bound, and u_j is the j th upper bound of problem variables.

The population members of pelicans in the proposed POA are identified using a matrix called the population matrix in Eq. (10). Each row of this matrix represents a candidate solution, while the columns of this matrix

represent the proposed values for the problem variables.

$$X = \begin{bmatrix} X_1 \\ \vdots \\ X_i \\ \vdots \\ X_N \end{bmatrix}_{N \times m} = \begin{bmatrix} x_{1,1} & \dots & x_{1,j} & \dots & x_{1,m} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ x_{i,1} & \dots & x_{i,j} & \dots & x_{i,m} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ x_{N,1} & \dots & x_{N,j} & \dots & x_{N,m} \end{bmatrix}_{N \times m} \quad (10)$$

Let X_i represent the i th pelican, and X represents the pelican population matrix. Each member of the population in the suggested POA is a pelican, which is a potential fix for the stated issue. Consequently, we can use each of the potential solutions to evaluate the objective function of the given problem. Eq. (11) uses a vector known as the objective function vector to derive the values obtained for the objective function.

$$F = \begin{bmatrix} F_1 \\ \vdots \\ F_i \\ \vdots \\ F_N \end{bmatrix}_{N \times 1} = \begin{bmatrix} F(X_1) \\ \vdots \\ F(X_i) \\ \vdots \\ F(X_N) \end{bmatrix}_{N \times 1} \quad (11)$$

where F_i is the objective function value of the i th candidate solution, and F is the objective function vector, To update potential answers, the suggested POA mimics pelicans' hunting and attack tactics and behavior. There are two stages to simulating this hunting strategy:

1. Approaching the prey (exploration phase).
2. Winging is done during the exploitation phase on the water's surface.

Phase 1: The Discovery Phase: Approaching the Prey The initial stage of the process involves the pelicans locating the prey and then approaching it. Modeling this pelican's approach enables it to scan the search space and explore various search space regions. The key to POA lies in randomizing the prey's position within the search space. As a result, POA has more exploration capacity while precisely searching the problem-solving domain. Eq. (12) simulates the aforementioned ideas as well as the pelican's analytical approach to the location of prey.

$$x_{i,j}^{P_1} = \begin{cases} x_{i,j} + \text{rand.} \cdot (p_j - I \cdot x_{i,j}), & F_p < F_i \\ x_{i,j} + \text{rand.} \cdot (x_{i,j} - p_j), & \text{else} \end{cases} \quad (12)$$

where p_j is the position of prey in the j th dimension, F_p is its objective function value, and $x_{i,j}^{P_1}$ is the new status of the i th pelican in the j th dimension based on phase 1. The parameter I is a number that can arbitrarily equal 1 or 2. We choose a random parameter for each member and iteration. A value of two for this parameter further displaces a member, potentially leading them into previously unexplored regions of the search space. As a result, parameter I influences the POA's ability to explore and precisely scan the search space. We accept the new location for a pelican if the suggested POA enhances the value of the objective function. This type of update, known as effective updating, keeps the algorithm from going into sub optimal regions. We model this process using an Eq. (13) [52].

$$X_i = \begin{cases} X_i^{P_1}, & F_i^{P_1} < F_i \\ X_i & \text{else} \end{cases} \quad (13)$$

where $F_i^{P_1}$ is the pelican's objective function value based on phase 1, and $X_i^{P_1}$ is its new status of the i th pelican.

Phase 2: The Exploitation Phase, or Winging on the Water Surface In the second stage, the pelicans gather their prey in their throat pouches after reaching the water's surface and spreading their wings to push the fish upward. This tactic helps pelicans catch more fish in the attacked region. As a result of simulating this pelican behavior, the suggested POA converges to more advantageous locations within the hunting region. This procedure

boosts POA's capacity for local search and exploitation. From a mathematical perspective, the algorithm must look at the points surrounding the pelican position in order to converge on an optimal solution. Eq. (14) mathematically simulates pelicans' hunting behavior.

$$x_{i,j}^{P2} = x_{i,j} + R \cdot \left(1 - \frac{t}{T}\right) \quad (14)$$

where $R \cdot \left(1 - \frac{t}{T}\right)$ is the neighborhood radius of $x_{i,j}$, t is the iteration counter, and T is the maximum number of iterations. Additionally, $x_{i,j}^{P2}$ is the updated status of the i th pelican in the j th dimension based on phase 2. The radius of the neighborhood of the population members to search locally near each member to converge to a better solution is represented by the coefficient $R \cdot \left(1 - \frac{t}{T}\right)$. This coefficient effectively enhances the POA exploitation power by bringing the population closer to the ideal global solution. Initially, a larger region surrounds each member due to the high value of this coefficient. The $R \cdot \left(1 - \frac{t}{T}\right)$ coefficient falls as the method replicates more, resulting in smaller radii for each member's neighborhood. This allows us to scan the region surrounding each member of the population in smaller and more precise steps, ensuring the POA converges to solutions that are closer to the global (and even precisely global) optimal based on the usage concept. Eq. (15) models the new pelican posture, which has also undergone effective updating and received acceptance or rejection.

$$X_i = \begin{cases} X_i^{P2}, & F_i^{P2} < F_i \\ X_i & \text{else} \end{cases} \quad (15)$$

where F_i^{P2} is the pelican's objective function value based on phase 2, and x_i^{P2} is its new status.

5. The proposed improving of POA

The Meta-heuristic algorithms are initially proposed, it was used mostly for continuous optimization problems. But mixed integer programming problems are not always problems with continuous variables, which means they are discrete variables. Binary optimization problems are a special case of discrete optimization, in which the search space variable is binary values [57, 58].

The transfer function plays a role in the optimizer binarization process for mapping the search space, optimizing the convergence speed and accuracy of the optimization algorithm. Under the transfer function, the position of the vector value is altered from 0 instead of 1 and vice versa. In this step, the transfer function will give a force to impel the particle in the binary space [59, 60].

Taking into consideration the critical need to use transfer functions to discretize the binary optimization problem, it makes sense to pay attention to the problem of discussing and exploring the design of transfer functions with regard to their diversity and effectiveness in practice. This being the case, the currently existing V-shaped transfer functions and S-shaped transfer functions [61, 62, 63] are initially revisited in this section.

In this paper we applied the new type of the family of transfer functions proposed by [64] which was known as tent-shaped transfer functions. According to this thought, a binary pelican optimization algorithm (BPOA) is suggested to mixed integer programming problems.

The V-shaped and S-shaped functions are the most common and widely used in the transfer functions [63]. The most representative V-shaped functions include V_1 to V_4 , and the most representative S-shaped functions include functions S1 to S4. The mathematical equations of these functions are presented in Table 1 and the curves of these functions are presented in Figure 1 and 2. They bring it out clear that S-shaped functions are a category of fundamental functions that are generated out of the exponential function.

The positions will be updated in the S-shaped functions using Eq. (16).

$$x_i^k(t+1) = \begin{cases} 1 & \text{if } r < S \text{ function } (x_i^k(t+1)) \\ 0 & \text{if } r \geq S \text{ function } (x_i^k(t+1)) \end{cases} \quad (16)$$

where r is a random number $\in [0, 1]$.

The positions will be updated in the V-shaped functions using Eq. (17).

$$x_i^k(t+1) = \begin{cases} x_i^k(t)^{-1} & \text{if } r < V \text{ function } (x_i^k(t+1)) \\ x_i^k(t) & \text{if } r \geq V \text{ function } (x_i^k(t+1)) \end{cases} \quad (17)$$

Table 1. Families of transfer functions

S-Shaped family		V-Shaped family	
S ₁	$S_1(x) = \frac{1}{1+e^{-2x}}$	V ₁	$V_1(x) = \left \operatorname{erf} \left(\frac{\sqrt{\pi}}{2}x \right) \right $
S ₂	$S_2(x) = \frac{1}{1+e^{-x}}$	V ₂	$V_2(x) = \tanh(x) $
S ₃	$S_3(x) = \frac{1}{1+e^{-\frac{x}{2}}}$	V ₃	$V_3(x) = \left \frac{x}{\sqrt{1+x^2}} \right $
S ₄	$S_4(x) = \frac{1}{1+e^{-\frac{x}{3}}}$	V ₄	$V_4(x) = \left \frac{2}{\pi} \arctan \left(\frac{\pi}{2}x \right) \right $

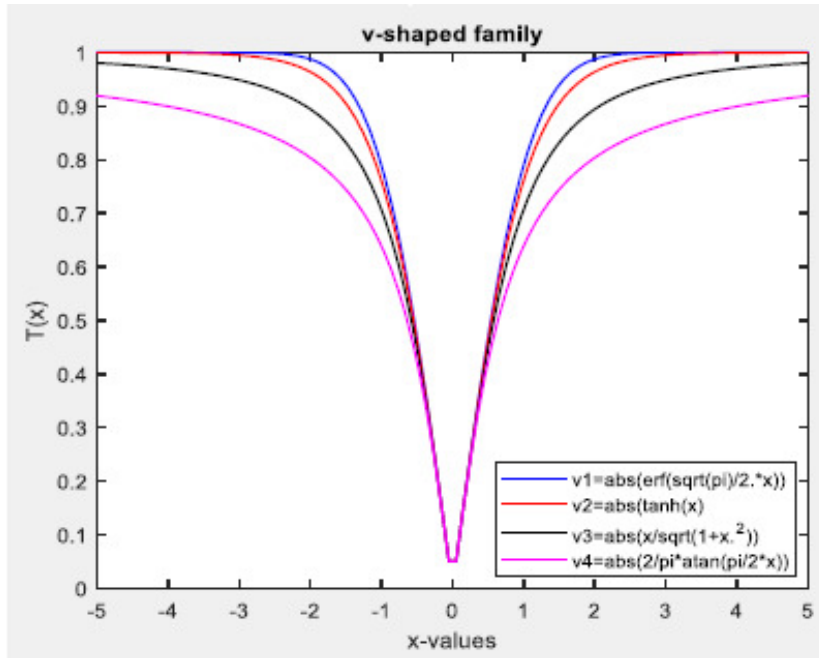


Figure 1. V-Shaped transfer function.

Today the existing transfer functions and their applicability [62, 65] are judged primarily by the results in binary optimization problems (BOPs). Resting on this fact, it becomes obvious that the attempts to develop a range of transfer functions can be quite helpful, not only does it offer more possibilities to discretize BOPs, but also it helps to gain more experience necessary to formulate a comprehensive theory that helps evaluate those functions. V-shaped transfer functions and S-shaped transfer functions [61] are composed of a trigonometric function, an anti-trigonometric function, an exponential function or a non-elementary function, thus, their computation needs more relative to a trigonometric function. This will affect the execution time of BOPs negatively. To this end, a new transfer function is suggested. Because the shape of the new transfer function is similar to the one of a tent, it can be referred to as a tent-shaped transfer function and is as follows [64]:

$$TT(x) = \frac{1}{(1+|x|)^R} \quad (18)$$

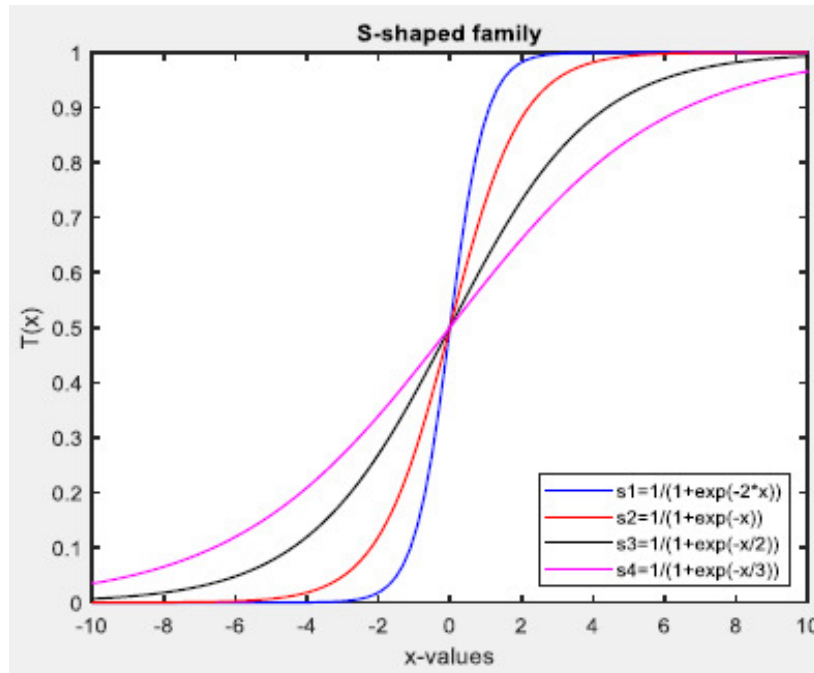


Figure 2. S-Shaped transfer function.

And R being a positive real number. With a suitable value of R a system of Tent functions family is obtained which have their expressions and Figures in Table 2 and Figure 3, respectively.

It is evident in Figures 2 and 3 that properties of the tent shaped functions are totally divergent to the S-shaped transfer functions. However, when we compare our function to the V-shaped transfer functions we find that they are only similar in that they both are symmetrical about the Y-axis, but they are quite different in a number of properties, including (but not limited to): The V-shaped transfer function is decreasing on $(-\infty, 0]$ increasing on $[0, \infty)$, and the reverse is true, it is obvious by looking at Figures 1 and 3 that the tent-shaped transfer functional is an inverted V-shaped transfer functional. This distinction might be insignificant compared with the following [64]:

1. All T-shaped transfer functions possess a unified mathematical formula. R represents a positive real number, and their computations are quite simple. Conversely, the V-shaped transfer functions lack a common calculation formula; furthermore, the mathematical formula for functions V_2 and V_4 are somewhat complicated.
2. The increase or decrease in the curvature of the T-shaped transfer functions is substantially more gradual than the curvature of the V-shaped transfer functions. This disparity in mathematical attributes is the most crucial element among the differing properties between T-shaped and V-shaped transfer functions.

Based on the two points above, the computational difficulty of T-shaped transfer functions is significantly less than that of V-shaped transfer functions.

6. Experimental Results and discussion

The improved mixed-integer pelican optimization algorithm (MI-POA) is applied in this section to eight integer and mixed-integer optimization problems. These problems are borrowed in various sources of the literature [66, 67, 68] and are shown in Table 3. And outline of these test problems is in the Appendix.

Table 2. T-Shaped transfer functions

Name	Transfer function
TT ₁	$TT_1(x) = \frac{1}{(1+ x)^{0.5}}$
TT ₂	$TT_2(x) = \frac{1}{(1+ x)}$
TT ₃	$TT_3(x) = \frac{1}{(1+ x)^2}$
TT ₄	$TT_4(x) = \frac{1}{(1+ x)^3}$

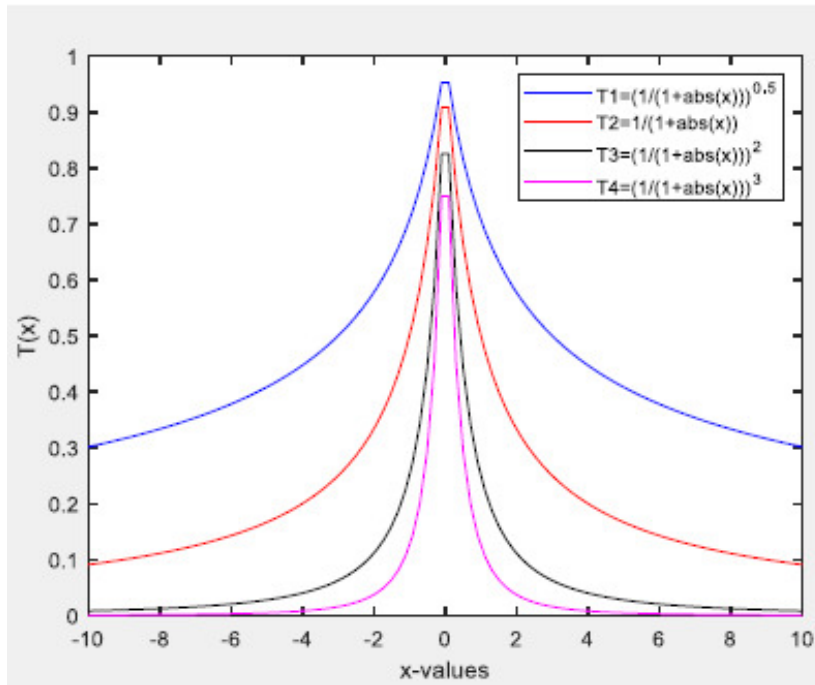


Figure 3. T-Shaped transfer function.

Table 3. Properties of the integer and mixed integer problems

Problems	Number of variables	Number of Integer variables	Number of constraints	Optimal
P ₁	2	1	2	2
P ₂	2	1	2	2.124
P ₃	3	1	3	1.07654
P ₄	4	4	1	-6
P ₅	3	1	4	99.245209
P ₆	7	4	9	3.557463
P ₇	8	8	4	0.94347
P ₈	5	3	4	7.667

In this paper, these problems have been addressed in the current paper using improved algorithm MI-POA. Overall, 100 runs of improved algorithm (MI-POA) of all the transfer functions considered in this paper have been performed on each problem. A successful run is defined as a successful run when the value of the error in the objective function value (error = |known objective function value - obtained objective function value|) is less than 0.01. This limit is assumed as in [67, 68]. All the experiments were carried out on MATLAB 2023. The parameters of the POA algorithm have been defined with their respective values in Table 4.

Tables 5 and 6 measure the performance of the improved algorithm in terms of some statistical measures: best, worst, mean and standard deviation of objective function values achieved in runs.

Table 4. Parameters of the POA algorithm

Parameter	Definition	Values
N	Population size	30
T	Maximum number of iterations	500
I	Randomly selected per member and iteration	1 or 2
R	Constant	0.2
t	Current iteration number	1 to T

Table 5. Results obtained by MI-POA for P_1, P_2, P_3, P_4

Problems	Transfer function	Best	Worst	Mean	S.D	
P ₁	S ₁	2	2.1644	2.0526	0.1228	
	S ₂	2	2.1673	2.0538	0.1294	
	S ₃	2	2.1755	2.0623	0.1317	
	S ₄	2	2.1691	2.0564	0.1309	
	V ₁	2	2.1382	2.0248	0.1043	
	V ₂	2	2.1361	2.0240	0.1040	
	V ₃	2	2.1358	2.0239	0.1033	
	V ₄	2	2.1332	2.0207	0.1011	
	TT ₁	2	2.0154	2.0066	0.0245	
	TT ₂	2	2.0151	2.0063	0.0219	
	TT ₃	2	2.0136	2.0048	0.0175	
	TT ₄	2	2.0147	2.0051	0.0182	
	P ₂	S ₁	2.124	2.1258	2.1246	0.0736
		S ₂	2.124	2.1254	2.1240	0.0721
S ₃		2.124	2.1261	2.1253	0.0740	
S ₄		2.124	2.1266	2.1259	0.0746	
V ₁		2.124	2.1250	2.1248	0.0440	
V ₂		2.124	2.12491	2.12431	0.04382	
V ₃		2.124	2.12485	2.12423	0.03821	
V ₄		2.124	2.12443	2.12411	0.03256	
TT ₁		2.124	2.12429	2.12403	0.02501	
TT ₂		2.124	2.12415	2.124005	0.02384	
TT ₃		2.124	2.12422	2.124009	0.02478	
TT ₄		2.124	2.12419	2.124003	0.02409	
P ₃		S ₁	1.076543	1.07671	1.07665	0.05201
		S ₂	1.0765427	1.0762	1.07659	0.05134
	S ₃	1.076544	1.07663	1.07660	0.05182	
	S ₄	1.076542	1.07649	1.076448	0.05112	
	V ₁	1.076543	1.07645	1.076346	0.0365	
	V ₂	1.076543	1.076309	1.07648	0.0409	
	V ₃	1.076543	1.07640	1.07642	0.0332	
	V ₄	1.076543	1.076399	1.076441	0.0301	
	TT ₁	1.076543	1.07648	1.076537	0.0041	
	TT ₂	1.076543	1.076396	1.07652	0.0032	
	TT ₃	1.076543	1.07644	1.07653	0.0038	
	TT ₄	1.076543	1.07637	1.076504	0.0033	
	P ₄	S ₁	-6.00	-6.00	-6.00	0
		S ₂	-6.00	-6.00	-6.00	0
S ₃		-6.00	-6.00	-6.00	0	
S ₄		-6.00	-6.00	-6.00	0	
V ₁		-6.00	-6.00	-6.00	0	
V ₂		-6.00	-6.00	-6.00	0	
V ₃		-6.00	-6.00	-6.00	0	
V ₄		-6.00	-6.00	-6.00	0	
TT ₁		-6.00	-6.00	-6.00	0	
TT ₂		-6.00	-6.00	-6.00	0	
TT ₃		-6.00	-6.00	-6.00	0	
TT ₄		-6.00	-6.00	-6.00	0	

Based on Tables 5 and 6, we observe that the P₁ all transfer functions have the same optimum value of 2, which proves that they find the optimum. S shaped transfer function (S₁ to S₄) have greater Means (~2.052-2.062) and

Table 6. Results obtained by MI-POA for P5, P6, P7, P8

Problems	Transfer function	Best	Worst	Mean	S.D	
P ₅	S ₁	99.24532	98.5331	98.6421	1.5207	
	S ₂	99.24525	98.5692	98.8734	1.5001	
	S ₃	99.24526	98.4783	98.7223	1.5034	
	S ₄	99.24525	98.4532	98.7112	1.5219	
	V ₁	99.24527	98.5953	98.8635	1.4712	
	V ₂	99.24524	98.86402	98.6655	1.3436	
	V ₃	99.24526	98.6123	98.4521	1.3076	
	V ₄	99.24522	98.8804	98.9232	1.2635	
	TT ₁	99.24521	98.8941	99.0536	1.0176	
	TT ₂	99.245219	98.8743	99.0722	1.0162	
	TT ₃	99.245206	98.9345	99.1089	1.0148	
	TT ₄	99.245208	98.9052	99.0922	1.0156	
	P ₆	S ₁	3.5577	4.4257	4.2516	0.3306
		S ₂	3.5578	4.5213	4.2342	0.3132
S ₃		3.5574	4.6375	4.3861	0.4222	
S ₄		3.5574	4.4503	4.4331	0.4365	
V ₁		3.5576	4.2366	3.6672	0.3225	
V ₂		3.5576	4.2401	3.6744	0.3318	
V ₃		3.5576	4.2397	3.6701	0.3274	
V ₄		3.5574	4.2065	3.6542	0.3129	
TT ₁		3.5575	3.9266	3.6325	0.2356	
TT ₂		3.5575	4.1632	3.6477	0.3004	
TT ₃		3.5575	4.1153	3.63002	0.2426	
TT ₄		3.5575	4.1467	3.6294	0.2495	
P ₇		S ₁	-0.94347	-0.92194	-0.93452	0.0267
		S ₂	-0.94347	-0.91543	-0.92231	0.0343
	S ₃	-0.94347	-0.90962	-0.93652	0.0285	
	S ₄	-0.94347	-0.92236	-0.93383	0.0225	
	V ₁	-0.94347	-0.92451	-0.93162	0.0174	
	V ₂	-0.94347	-0.93272	-0.93352	0.00952	
	V ₃	-0.94347	-0.92924	-0.93248	0.0166	
	V ₄	-0.94347	-0.93152	-0.93227	0.00981	
	TT ₁	-0.94347	-0.93576	-0.93562	0.00632	
	TT ₂	-0.94347	-0.93887	-0.93874	0.00357	
	TT ₃	-0.94347	-0.93795	-0.93836	0.00442	
	TT ₄	-0.94347	-0.93842	-0.93982	0.00318	
	P ₈	S ₁	7.66792	7.28422	7.45561	1.6427
		S ₂	7.66785	7.3288	7.4472	1.5113
S ₃		7.66788	7.3433	7.4396	1.3371	
S ₄		7.667911	7.2458	7.4175	1.6142	
V ₁		7.66784	7.4428	7.6639	0.9767	
V ₂		7.66789	7.4954	7.5822	1.0123	
V ₃		7.66784	7.4593	7.6651	0.9422	
V ₄		7.66784	7.4674	7.6659	0.9351	
TT ₁		7.66784	7.5322	7.6662	0.4591	
TT ₂		7.66781	7.5643	7.6659	0.2266	
TT ₃		7.66783	7.5778	7.66578	0.4673	
TT ₄		7.667809	7.5769	7.6669	0.3228	

S.D (~0.123-0.132), which represents greater variability. V shaped transfer function are slightly better (Means = -2.021 -2.025, S.D = -0.101-0.104), and TT shaped transfer function are dominant with Means (= -2.0048 - 2.0066) and lowest S.D (= -0.0175 -0.0245). TT transfer functions show better stability of P₁ with TT₃ showing the narrowest spread (Worst=2.0136, S.D=0.0175).

With respect to P₂ all transfer functions (S₁-S₄, V₁-V₄, TT₁-TT₄) give the precise Best value of 2.124, which is consistent optimum discovery. S shaped transfer function exhibits Means of 2.1240-2.1259 and greater S.D (0.0721-0.0746), which indicates moderate instability. V shaped transfer function narrows to Means 2.12411-2.1248 with S.D 0.0326-0.0440, whereas TT shaped transfer function performs better at Means 2.12400-2.12403

and minimum S.D (0.0238-0.0248), headed by TT₂, TT₄ (Worst=2.12415-2.12419). TT shaped transfer functions dominate P₂ with high precision and low variance, which is suitable in metaheuristic applications in mixed-integer.

Also, we observe that all transfer functions of P₃ hit the very Best value (1.076543 or minor variants such as 1.0765427) but S shaped transfer functions have higher Means (~1.07645-1.07665) and S.D (~0.051-0.052) which are variable. V shaped transfer functions are better (Means = -1.07635-1.07648, S.D=0.030-0.041) and TT shaped transfer functions are shining (Means = -1.07650-1.07654, S.D=0.003-0.004), TT₂ being the best (Worst=1.076396, S.D=0.0032).

In the case of the P₄, all transfer functions (S₁-S₄, V₁-V₄, TT₁-TT₄) have the same Best, Worst, Mean (-6.00), and S.D (0) values, indicating complete convergence to zero variability between runs. This homogeneity establishes of problem P₄ trivial nature, where algorithms trivially achieve the optimum without going wrong.

From Table 6 concerning the P₅, all transfer functions reach Best values close to 99.245 but S shaped transfer function exhibit Means close to 98.64-98.87 with higher S.D (close to 1.50-1.52) indicating variability. V shaped transfer function improve to Means =98.45-98.92, S.D =1.26-1.47, TT shaped transfer function dominate with Means = 99.05-99.11, lowest S.D (=1.01-1.02) dominated by TT₃ (Worst=98.9345).

Also the P₆ we see that all transfer functions converge to Best values around 3.557, however, S shaped transfer function converge to higher Means (4.23-4.43) and S.D (~0.31-0.44), and hence worse convergence. V shaped transfer function is much better (Means =3.65-3.67, S.D =0.31-0.33), whereas TT shaped transfer function dominates with Means =3.63-3.65, and smaller S.D (=0.24-0.30), especially TT₁ (Worst=3.9266).

In the P₇, all the transfer functions give the same Best value of -0.94347, but S shaped transfer functions give Means of -0.922 to -0.936 with a larger S.D (~0.023-0.034), which is a measure of variability. V shaped transfer function narrows to Means -0.931 -0.933 (S.D 0.0095-0.017) whereas TT shaped transfer function performs best with Means -0.935 -0.939 and smallest S.D (0.003-0.006) with TT₂ and TT₄ leading the way (Worst near -0.938).

With regard to the P₈, All transfer functions are Best values in the range of 7.6678-7.6679, but S shaped transfer functions have the highest Means (7.4175-7.4556) and largest S.D (~1.34-1.64), which implies a high level of variability and worse convergence. V shaped transfer function improve to Means =7.58-7.666 (S.D=0.94-1.01), and TT shaped transfer function dominate with Means=7.665-7.667 and lowest S.D=(~0.23-0.47), and TT₂ led (Worst=7.5643, S.D=0.227).

Overall, TT shaped transfer function dominance has the tightest consistency, zero S.D on P₄, approximate 0.003-0.02 on problems (P₂-P₃/P₇), and minimum spreads of approximately 0.2-0.5 on P₅-P₆/P₈. V shaped transfer function second with moderate betterment than S shaped transfer function and S shaped transfer function exhibits most variability (S.D =0.05-1.6) which is best in exploration but not precision. Table 7 explains best transfer function , lowest S.D and highest S.D of every problems, and figure 4 explain standard deviation with each problem.

Table 7. best transfer function, lowest S.D and highest S.D of every problems

Problem	Best transfer function	Lowest S.D	Highest S.D
P1	TT (0.0175)	TT (0.0175-0.0245)	S (0.1228-0.1317)
P2	TT/ (~0.024)	TT (0.0238-0.0250)	S (0.0721-0.0746)
P3	TT (0.0032)	TT (0.0032-0.0041)	S (0.0511-0.0520)
P4	All (0)	All (0)	All (0)
P5	TT (1.0148)	TT (1.0148-1.0176)	S (1.5001-1.5219)
P6	TT (0.2356)	TT (0.2356-0.3004)	S (0.3132-0.4365)
P7	TT (0.0036)	TT (0.0032-0.0063)	S (0.0225-0.0343)
P8	TT (0.2266)	TT (0.2266-0.4673)	S (1.3371-1.6427)

Based on Figure 4, the TT-Shaped transfer function have the lowest standard deviation in all problems, which implies a high convergence irrespective of the initialization.

Figures 5-6 show the performance of S-Shaped transfer function , V-Shaped transfer function and TT-Shaped transfer function in terms of time that represents the average computation time in seconds used by the all algorithms in the case of successful runs. We can see how the TT-Shaped transfer function is better than the S-Shaped transfer function and V-Shaped transfer function.

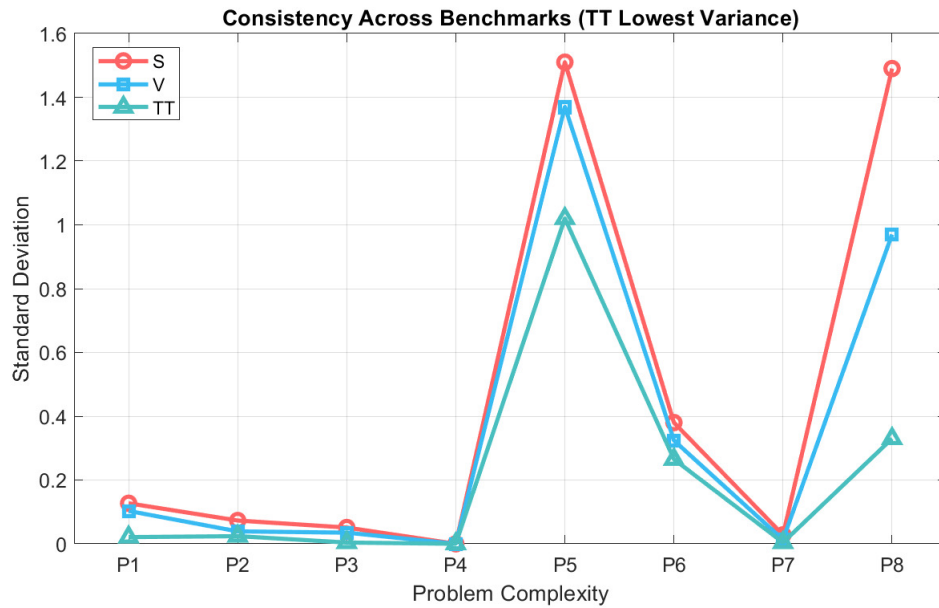


Figure 4. S.D values of each Problem.

In order to further confirm the performance of MI-POATT₄ with two evolutionary algorithms are compared: Black hole algorithm (BHA) and Flower pollination algorithm (FPA). Experimental results obtained by three algorithms are given in Table 8, where (PS) is success rate representing percentage of the successful runs to total runs for the algorithm, and (FE) represents average number of objective function evaluations used by the algorithm in successful runs. According to the results in Table 8, it is clear that MI-POATT₄ algorithm can obtain the optimal solution to different kinds of linear or nonlinear mixed integer programming problems with a success rate of 100 percent of all test functions except problem 6 with 92 %, which proves the stability and robustness of MI-POATT₄. in order to measure the efficiency of MI-POATT₄ in solving all test problems, the sum of the mean FE provided by BHA and FPA algorithms are 36439 and 28729 while the sum of the FE produced by MI-POA is 8321. In this way, MI-POA is considered to be the most effective and with the best minimum total (FE). Figure 7 and 8 represent the performance MI-POATT₄, BHA and FPA in terms of total function evaluation and total number of success of all test functions. Overall, MI-POATT₄ is better than any of the compared EAs algorithms regarding the measure of success rate and efficiency.

Table 8. Results obtained by three algorithms

Problems	MI-POA TT ₄			BHA			FPA		
	FE	PS	t	FE	PS	t	FE	PS	t
P1	330	100	0.0402	256	76	1.3124	198	83	2.5412
P2	140	100	0.0697	130	74	1.3767	567	65	2.7311
P3	1572	100	0.2822	4487	69	3.6477	5437	77	2.8932
P4	133	100	0.0053	375	100	0.8479	120	100	1.2184
P5	2421	100	0.7448	12450	65	2.3674	6387	72	1.7752
P6	4630	92	1.4523	8576	55	5.7312	3746	67	7.3444
P7	800	100	1.7201	687	77	4.2287	7644	100	3.0276
P8	225	100	0.5541	488	100	3.3651	430	100	6.1067

A statistical analysis is conducted to further evaluate the performance of the TT4 method. The Wilcoxon signed-rank test is a non-parametric statistical test. We use the Wilcoxon signed-rank test (WST) to find the important difference between TT4 and the other methods that were talked about. The WST results are presented in Table 9, with a significance level of $\alpha = 0.05$. As shown in Table 9, the statistical significance of the proposed method is better than that of other methods and the p-value is less than 0.05 for all methods.

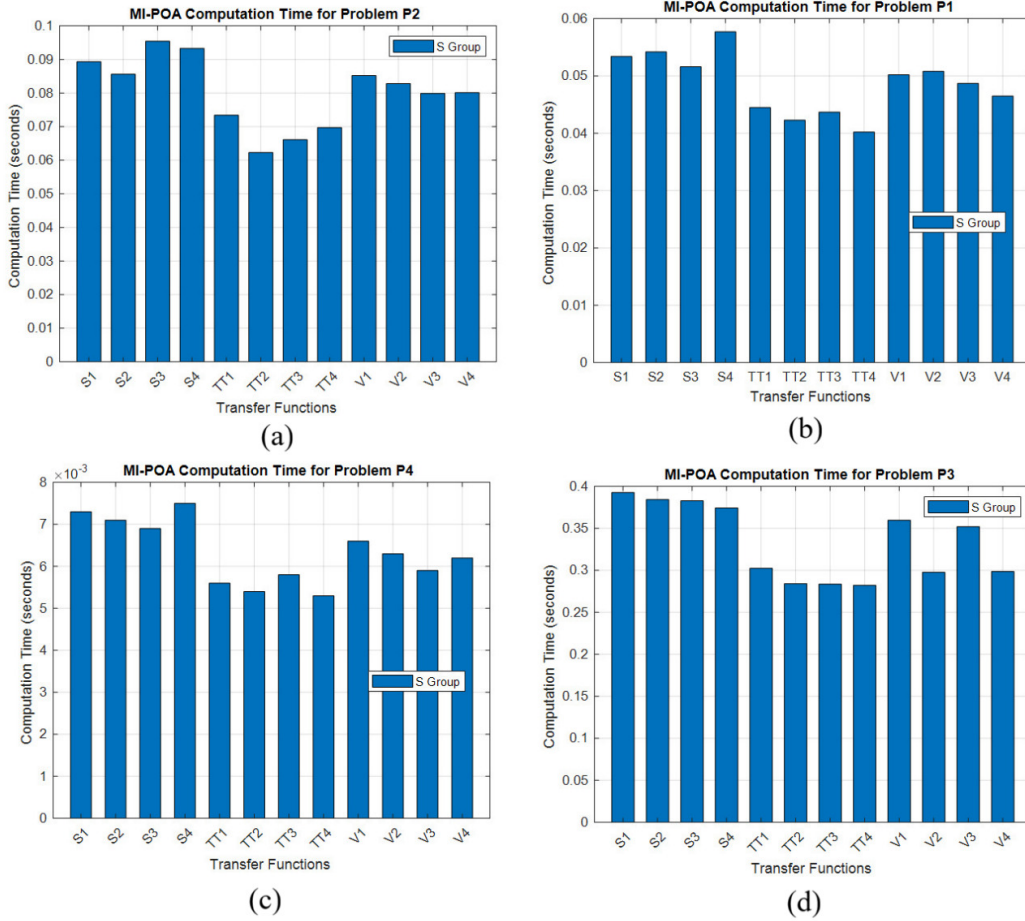


Figure 5. computational time (a) P1 , (b) P2 , (c) P3 , and (d) P4.

Table 9. p-values for the Wilcoxon signed-rank test of the proposed method results with three competitor methods

Pairwise comparison	p-value
TT4 vs S1	0.0025
TT4 vs S2	0.0021
TT4 vs S3	0.0034
TT4 vs S4	0.0027
TT4 vs V1	0.0025
TT4 vs V2	0.0028
TT4 vs V3	0.0014
TT4 vs V4	0.0027

7. Conclusion

This study successfully developed and validated MI-POA, an enhanced OPA tailored for integer and mixed-integer linear programming problems, by integrating novel tent-shaped transfer functions that outperform traditional S- and V-shaped alternatives in discretization efficiency and solution precision. Experimental results across eight benchmark problems confirm MI-POA’s superior performance, with tent functions (TT1-TT4) achieving the lowest standard deviations such as 0.0032 for P3, means closest to optima, such as 99.1089 for P5, and perfect optimality on simpler cases, such as P4 at -6.00, demonstrating robust exploration-exploitation balance and computational

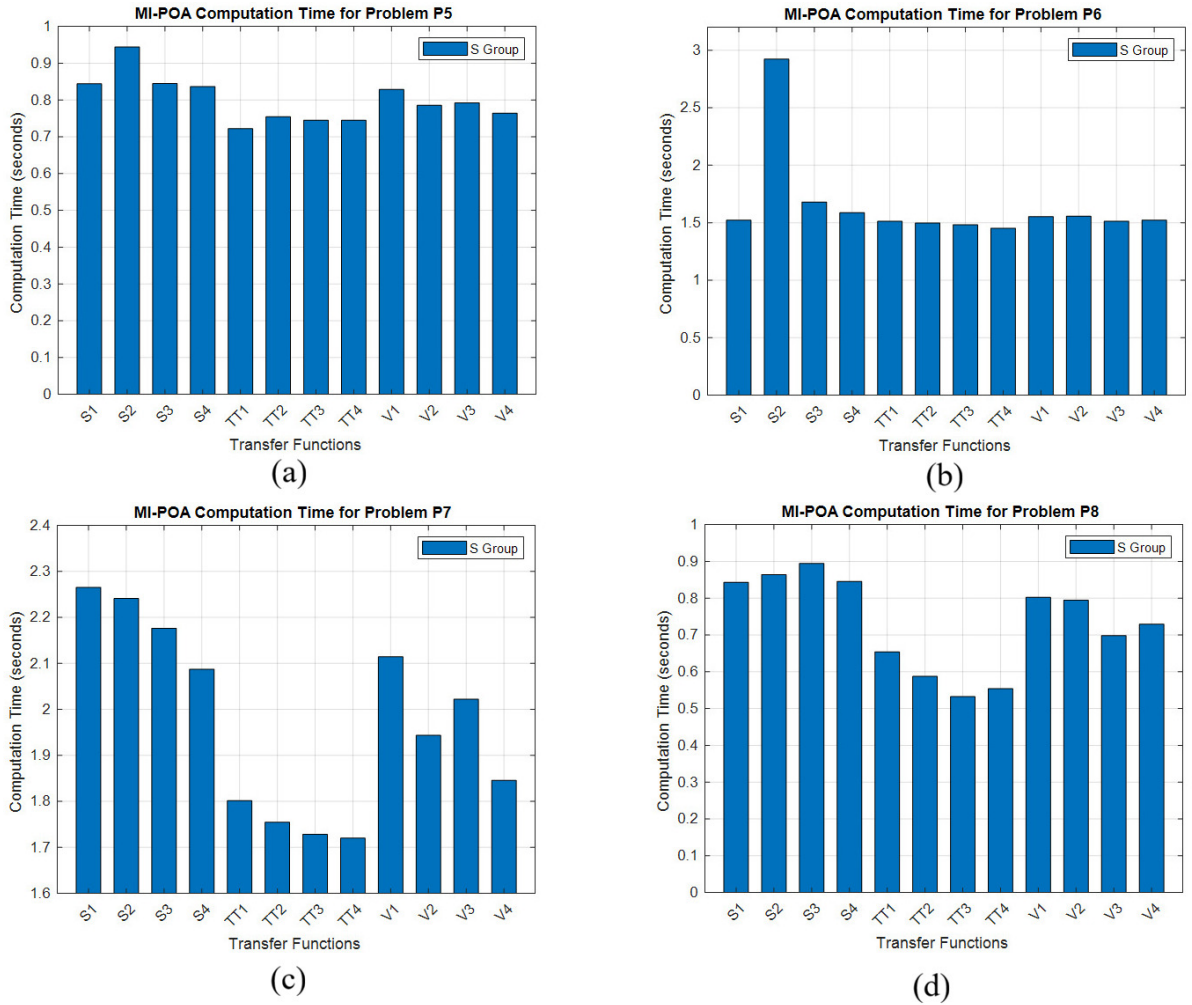


Figure 6. computational time of (a) P5 , (b) P6 , (c) P7 , and (d) P8.

simplicity. MI-POA advances metaheuristic solvers for NP-hard real-world applications in scheduling, logistics, and resource allocation, offering a promising, scalable alternative to exact methods like branch-and-bound. Future work will explore hybridizations and large-scale implementations.

Appendix

Problem (I)

$$\text{minf} = 2x + y$$

subject to:

$$1.25 - x^2 - y \leq 0$$

$$x + y \leq 1.6$$

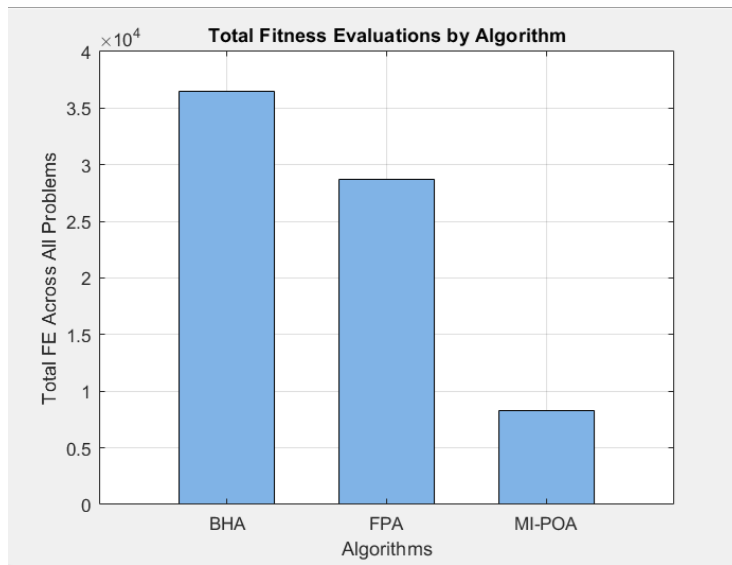


Figure 7. The performance MI-POA TT_4 , BHA and FPA in terms of total function evaluation



Figure 8. The performance MI-POA TT_4 , BHA and FPA in terms of total number of success

$$0 \leq x \leq 1.6$$

$$y \in \{0, 1\}$$

The optimal solution is:

$$f = 2, (x, y) = (0.5, 1)$$

Problem (2)

$$\min f = -y + 2x - \ln\left(\frac{x}{2}\right)$$

subject to:

$$\begin{aligned} -x - \ln\left(\frac{x}{2}\right) + y &\leq 0 \\ 0.5 &\leq x \leq 1.5 \\ y &\in \{0, 1\} \end{aligned}$$

The optimal solution is:

$$f = 2.124, (x, y) = (1.375, 1)$$

Problem (3)

$$\text{minf} = -0.7y + 5(x_1 - 0.5)^2 + 0.8$$

subject to:

$$\begin{aligned} -\exp(x_1 - 0.2) - x_2 &\leq 0 \\ x_2 + 1.1y &\leq -1 \\ x_1 - 1.2y &\leq 0.2 \\ 0.2 &\leq x_1 \leq 1 \\ -2.22554 &\leq x_2 \leq -1 \\ y &\in \{0, 1\} \end{aligned}$$

The optimal solution is:

$$f = 1.07654, (x_1, x_2, y) = (0.94194, -2.1, 1)$$

Problem (4)

$$\text{minf} = (x_1 + 2x_2 + 3x_3 - x_4)(2x_1 + 5x_2 + 3x_3 - 6x_4)$$

subject to:

$$\begin{aligned} x_1 + 2x_2 + x_3 + x_4 &\leq 4 \\ x_i &\in \{0, 1\}; i = 1, 2, 3, 4 \end{aligned}$$

The optimal solution is:

$$f = -6, (x_1, x_2, x_3, x_4) = (0, 0, 1, 1)$$

Problem (5)

$$\text{minf} = 7.5y + 5.5(1-y) + 7v_1 + 6v_2 + 50 \frac{y/(2y-1)}{0.9[1-\exp(-0.5v_1)]} + 50 \frac{1 - (y/(2y-1))}{0.8[1-\exp(-0.4v_2)]}$$

subject to:

$$\begin{aligned} 0.9[1-\exp(-0.5v_1)] - 2y &\leq 0 \\ 0.8[1-\exp(-0.4v_1)] - 2(1-y) &\leq 0 \\ v_1 &\leq 10y \\ v_2 &\leq 10(1-y) \\ v_1, v_2 &\geq 0 \\ y &\in \{0, 1\} \end{aligned}$$

The optimal solution is:

$$f = 99.245209, (y, v_1, v_2) = (1, 3.514237, 0)$$

Problem (6)

$$\min f = (y_1 - 1)^2 + (y_2 - 1)^2 + (y_3 - 1)^2 - \ln(y_4 + 1) + (x_1 - 1)^2 + (x_2 - 2)^2 + (x_3 - 3)^2$$

subject to:

$$y_1 + y_2 + y_3 + x_1 + x_2 + x_3 \leq 5$$

$$y_3^2 + x_1^2 + x_2^2 + x_3^2 \leq 5.5$$

$$y_1 + x_1 \leq 1.2$$

$$y_2 + x_2 \leq 1.8$$

$$y_3 + x_3 \leq 2.5$$

$$y_4 + x_1 \leq 1.2$$

$$y_2^2 + x_2^2 \leq 1.64$$

$$y_3^2 + x_3^2 \leq 4.25$$

$$y_2^2 + x_3^2 \leq 4.64$$

$$x_i \geq 0 ; i = 1, 2, 3$$

$$y_i \in \{0, 1\} ; i = 1, 2, 3, 4$$

The optimal solution is:

$$f = 3.557463, (x_1, x_2, x_3, y_1, y_2, y_3, y_4) = (0.2, 1.280624, 1.954483, 1, 0, 0, 1)$$

Problem (7)

$$\min f = r_1 r_2 r_3$$

$$\text{where } r_1 = 1 - 0.1^{y_1} 0.2^{y_2} 0.15^{y_3}$$

$$r_2 = 1 - 0.05^{y_4} 0.2^{y_5} 0.15^{y_6}$$

$$r_3 = 1 - 0.02^{y_7} 0.06^{y_8}$$

subject to:

$$y_1 + y_2 + y_3 \geq 1$$

$$y_4 + y_5 + y_6 \geq 1$$

$$y_7 + y_8 \geq 1$$

$$3y_1 + y_2 + 2y_3 + 3y_4 + 2y_5 + y_6 + 3y_7 + 2y_8 \leq 10$$

$$y_i \in \{0, 1\} ; i = 1, 2, 3, 4, 5, 6, 7, 8$$

The optimal solution is:

$$f = -0.94347, (y_1, y_2, y_3, y_4, y_5, y_6, y_7, y_8) = (0, 1, 1, 1, 0, 1, 1, 0)$$

Problem (8)

$$\text{min } f = 2x_1 + 3x_2 + 1.5y_1 + 2y_2 - 0.5y_3$$

subject to:

$$x_1^2 + y_1 = 1.25$$

$$x_2^{1.5} + 1.5y_2 = 3$$

$$x_1 + y_1 \leq 1.6$$

$$1.333x_2 + y_2 \leq 3$$

$$0 \leq x_i \leq 2; i = 1, 2$$

$$y_j \in \{0, 1\}; j = 1, 2, 3$$

The optimal solution is:

$$f = 7.667, (x_1, x_2, y_1, y_2, y_3) = (1.117, 1.310, 0, 1, 1)$$

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