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Performance Sharia-Compliant Portfolio Based on Frank-Wolfe Optimization in The Indonesian Capital Market

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Abstract Portfolio weight optimization is a fundamental step in constructing efficient investment portfolios. In Islamic finance, however, portfolio construction must satisfy Sharia principles that prohibit short selling, interest-based transactions, excessive uncertainty, and speculative behavior. These constraints restrict the applicability of conventional portfolio optimization approaches and present challenges in achieving diversification and stable risk-return performance. This study investigates the construction of Sharia-compliant portfolios using stocks from the Jakarta Islamic Index (JII) within a mean-variance optimization framework. Portfolio allocations are obtained under long-only constraints using the Frank-Wolfe algorithm, whose projection-free structure naturally enforces non-negativity of portfolio weights. The analysis is conducted using daily return data from 2020 to 2024, considering portfolios with different numbers of assets and varying levels of risk aversion. The empirical results show that optimized Sharia-compliant portfolios achieve competitive risk-return performance relative to the Indonesian composite stock index (JKSE). Increasing the number of assets improves diversification and reduces portfolio volatility, while the risk-aversion parameter provides systematic control over portfolio allocation and risk exposure. The Frank-Wolfe algorithm demonstrates stable convergence and produces solutions consistent with those obtained from a quadratic programming benchmark. These findings indicate that portfolio optimization within the Sharia-compliant investment universe is both feasible and practically relevant. The proposed framework provides a computationally efficient approach for constructing diversified Islamic portfolios while respecting investment constraints imposed by Sharia principles.

Keywords No Short Selling; Positive Weights; Frank-Wolfe Algorithm; Indonesian Capital Market; Sharia-Compliant Portfolio

AMS 2010 subject classifications 62P20, 90C25, 90C06, 91G10, 65K05.

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1. Introduction

Islamic economics, grounded in the principles of the Qur'an and Sunnah, emphasizes ethical conduct, fairness, and social welfare in financial activities. Central to this framework are the prohibitions against *riba* (interest), *gharar* (excessive uncertainty), and *maysir* (gambling), which collectively shape the structure of Shariah-compliant financial systems [1]. These principles have evolved from classical Islamic economic thought into a modern institutional framework governing Islamic banking, capital markets, and investment practices, supported by international standard-setting bodies such as the Accounting and Auditing Organization for Islamic Financial Institutions (AAOIFI) and the Islamic Financial Services Board (IFSB).

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In Indonesia, the world's largest Muslim-majority country, the development of Islamic finance has been particularly robust. Regulatory oversight by the Financial Services Authority (OJK) and the National Sharia Board (DSN-MUI) has facilitated the growth of Shariah-compliant capital market instruments, including equity indices such as the Jakarta Islamic Index (JII) and the Sharia Stock Price Index (ISSI). These indices serve as benchmarks for investors seeking to balance financial performance with adherence to Islamic ethical principles [5]. Despite this progress, portfolio construction in Islamic finance remains methodologically constrained, as most classical portfolio optimization models implicitly permit short selling and unrestricted portfolio weights, which are inconsistent with Shariah requirements.

The Markowitz mean-variance framework, while foundational to modern portfolio theory, often generates optimal solutions involving negative asset weights, which are mathematically equivalent to short selling [6, 7]. Such practices violate the Shariah requirement that assets must be owned prior to sale, as reflected in the hadith prohibiting transactions involving assets not in one's possession [3]. Consequently, Muslim investors face challenges in achieving diversification and risk efficiency while maintaining full compliance with Islamic law. To address these limitations, recent studies have explored alternative optimization approaches, including heuristic methods and convex optimization techniques, to accommodate explicit no-short-selling constraints [8].

Among these approaches, the Frank-Wolfe algorithm (also known as the conditional gradient method) offers distinctive theoretical and computational advantages for Shariah-compliant portfolio optimization. Beyond its ability to enforce non-negativity constraints, the Frank-Wolfe algorithm is inherently projection-free and maintains feasibility at every iteration by generating portfolio updates as convex combinations of admissible asset allocations. This mechanism ensures that all intermediate and final portfolio weights remain within the Shariah-compliant feasible region. Moreover, the reliance on linear subproblems aligns with Islamic finance principles emphasizing asset ownership, avoidance of speculative leverage, and restriction to fully owned positions. In contrast to projection-based convex optimization methods, which may temporarily produce infeasible or non-compliant solutions, the Frank-Wolfe algorithm provides a theoretically consistent and computationally efficient framework for portfolio optimization under Shariah constraints [9].

Parallel to methodological developments, empirical research in Islamic finance has examined the behavior and performance of Shariah-compliant equities under various market conditions. Several studies report that Islamic stocks exhibit lower sensitivity to interest rate fluctuations, suggesting partial decoupling from conventional monetary shocks [10]. However, contrasting evidence indicates that Islamic equities may still be affected by interest rate movements, implying that Shariah-compliant markets are not entirely insulated from conventional financial dynamics [11]. Behavioral and cultural factors have also been shown to influence Islamic markets, including the documented Ramadan effect, during which stock returns tend to increase due to positive investor sentiment [12, 13]. Additionally, empirical findings suggest that Islamic equities may demonstrate greater resilience during financial crises due to the exclusion of highly leveraged and speculative sectors, although results vary across time periods and markets [14, 15].

Further evidence highlights the diversification and hedging potential of Islamic stocks. It has been shown that Islamic equities respond asymmetrically to market news, being less affected by adverse shocks than their conventional counterparts [16]. Other studies report that Islamic markets exhibit distinct behavior across market regimes and tend to display lower volatility during periods of financial distress [17, 18]. Collectively, these findings underscore the unique structural, behavioral, and regulatory characteristics of Shariah-compliant investments, reinforcing the need for specialized analytical and optimization frameworks.

Portfolio optimization in Islamic finance must comply with Sharia principles, which prohibit interest-based income, excessive uncertainty (gharar), and short selling. These constraints significantly affect feasible portfolio construction and risk management strategies. Recent studies have explored alternative optimization frameworks for Sharia-compliant portfolios. Lim et al. (2023) propose a CVaR-based Sharia-compliant portfolio model and demonstrate that diversification benefits remain achievable within constrained asset universes. Their findings indicate that downside-risk measures may offer improved robustness under Islamic investment rules.

In the Indonesian context, Qudratullah et al. (2025) develop a Shariah-Compliant Asset Pricing Model (SCAPM) applied to Jakarta Islamic Index (JII) stocks, showing that Sharia-compliant portfolios can deliver competitive risk-adjusted performance relative to conventional benchmarks. Similarly, Sales (2024) examines Sharia portfolio optimization in emerging markets and confirms that constrained optimization remains viable in practice.

Comparative empirical analyses, such as Asl et al. (2024), further investigate whether Islamic investing alters portfolio performance characteristics. Their findings suggest that while performance differences exist, diversification and systematic optimization remain critical determinants of risk-adjusted returns. These studies highlight the growing relevance of constrained portfolio optimization in Islamic capital markets and motivate the need for computationally efficient solution methods tailored to Sharia-compliant settings. The mean–variance portfolio problem under long-only constraints forms a convex quadratic optimization problem over the probability simplex. While quadratic programming (QP) solvers provide exact solutions, projection-based methods may become computationally demanding in high-dimensional settings.

The Frank–Wolfe (FW) algorithm, originally introduced by Frank and Wolfe (1956), has recently gained renewed attention in constrained convex optimization. Pokutta (2024) provides a modern overview of projection-free optimization methods, emphasizing their efficiency when the feasible set admits simple linear minimization oracles. Applied research has also demonstrated the practical relevance of FW in financial optimization. Setyawan (2025) proposes an improved Frank–Wolfe scheme for tactical portfolio allocation and shows that projection-free methods can approximate QP solutions with lower computational overhead. Motivated by these developments, this study evaluates the suitability of the Frank–Wolfe algorithm for Sharia-compliant portfolio optimization and benchmarks its performance against a standard quadratic programming solver.

Prior optimization studies involving Indonesian Islamic equity indices have largely relied on traditional mean-variance models or heuristic approaches, often without explicitly embedding no-short-selling constraints within the optimization process [36, 19]. As a result, there is limited empirical evidence on the effectiveness of modern convex optimization techniques in balancing return maximization, risk minimization, and strict Shariah compliance.

Despite the growing body of research on Islamic portfolio construction, limited empirical work has systematically evaluated diversification effects across varying portfolio sizes under Sharia constraints while simultaneously assessing computational efficiency of projection-free optimization methods in this context.

This study contributes to the literature in three ways. First, it provides a systematic out-of-sample evaluation of Sharia-compliant portfolio optimization in the Indonesian Islamic equity market using multiple portfolio sizes, thereby addressing concerns related to limited asset universes in prior studies. Second, it offers an economically interpretable sensitivity analysis across varying levels of investor risk aversion, illustrating how preference parameters influence diversification and allocation stability under Sharia constraints. Third, it evaluates the suitability of the projection-free Frank–Wolfe algorithm for long-only Islamic portfolio optimization and benchmarks its solutions against an exact quadratic programming solver, demonstrating convergence behavior and numerical consistency. Together, these elements position the study at the intersection of empirical Islamic finance and computational portfolio optimization.

2. Methodology

2.1. Data

The data used in this study were obtained from official publications and secondary sources provided by the Indonesian Financial Services Authority (OJK), the Indonesia Stock Exchange (IDX), and the historical market database of the Jakarta Islamic Index (JII). These sources ensure data reliability and consistency for empirical analysis in the Indonesian Islamic capital market.

The Jakarta Islamic Index (JII) serves as the primary benchmark for Shariah-compliant equities in Indonesia. The index consists of 45 liquid and large-cap Shariah-compliant stocks listed on the IDX and is reviewed semiannually to ensure ongoing compliance with Shariah screening criteria. Complementary Islamic equity indices include the Sharia Stock Price Index (ISSI), the Jakarta Islamic Mid-Cap Index (JIMI), and the Jakarta Islamic Small-Cap Index (JISML), reflecting the breadth of Shariah-compliant equities available in the Indonesian market [20, 21]. In this study, JII is selected as the investment universe due to its liquidity, representativeness, and relevance for portfolio construction under Shariah constraints.

Figure 1 illustrates the long-term evolution of trading volume in the JII and the Indonesian Composite Index (IHSG) or Jakarta Composite Stock Exchange (JKSE) from 2003 to 2024. The figure provides contextual evidence of increasing market participation in both Shariah-compliant and conventional equity segments. Although aggregate index-level trading volume is not directly used in the portfolio optimization process, this comparison motivates the selection of JII as a representative universe for empirical analysis in Islamic portfolio optimization.

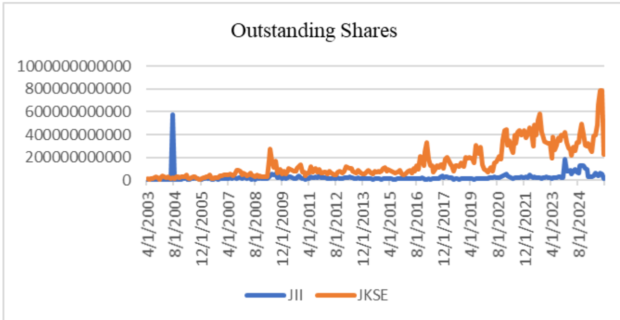

Figure 1. Annual trading volume of JII and JKSE (2003–2024).

Figure 2 compares the cumulative returns of JII with major Indonesian equity indices, namely JKSE, IDX30, and LQ45, over the period 2014–2024. The comparison provides a broader performance context for Shariah-compliant equities relative to conventional benchmarks. While JII exhibits lower cumulative returns than some conventional indices, its volatility pattern remains comparable, supporting its suitability as a basis for portfolio modeling under Shariah screening constraints.

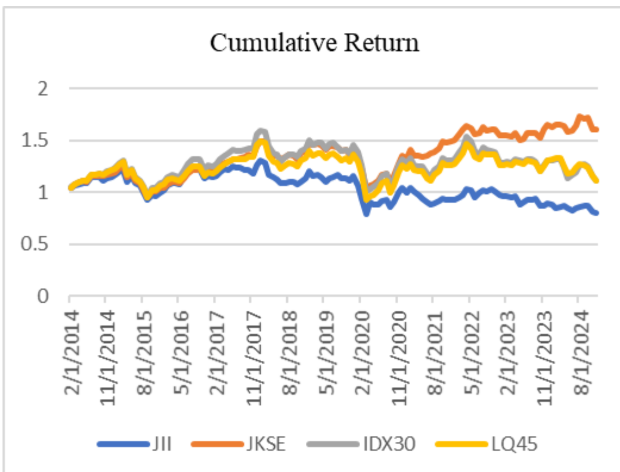

Figure 2. Cumulative returns of JII, JKSE, IDX30, and LQ45 (2014–2024). Source: Yahoo Finance, 2025.

This study focuses on a subset of four Shariah-compliant equities selected from the JII to construct the investment portfolio. The selection is based on inter-asset correlation analysis to enhance diversification and reduce unsystematic risk. Monthly return correlations were computed for all JII constituents over the period January 2020 to November 2025.

Price data for all selected stocks were collected from the official IDX website and Yahoo Finance, consisting of monthly adjusted closing prices from January 2020 to November 2025. Monthly logarithmic returns were computed as:

$$R_t = \ln\left(\frac{P_t}{P_{t-1}}\right) \tag{1}$$

where P_t and P_{t-1} denote the adjusted closing prices at time t and $t-1$, respectively. The use of monthly data mitigates short-term noise while preserving long-term return dynamics, consistent with prior studies on Shariah-compliant portfolio optimization [36, 5, 22].

To address potential selection bias, the analysis is conducted across multiple portfolio sizes (n = 5, 10, 15, 20) drawn from consistently listed JII constituents. This approach mitigates dependence on a specific subset and allows robustness assessment across varying asset universes. While the JII itself contains a limited number of screened stocks, the use of multiple configurations ensures that the results are not driven by a particular stock selection.

All computations were conducted using Python 3.12, with NumPy and Pandas libraries employed for numerical and statistical processing. The resulting dataset provides a reliable empirical foundation for the subsequent Frank-Wolfe optimization analysis, enabling an effective examination of the risk-return trade-off under explicit Shariah constraints in Indonesia's Islamic capital market.

2.2. *Markowitz Portfolio Model*

The Markowitz portfolio model, also known as Modern Portfolio Theory (MPT), is a foundational framework in quantitative investment analysis. Introduced by Harry Markowitz in 1952, the model focuses on constructing portfolios that maximize expected return for a given level of risk, or equivalently, minimize risk for a given level of expected return. The core principle underlying MPT is diversification, whereby combining assets with different risk-return characteristics can reduce overall portfolio risk without proportionally reducing expected returns.

Markowitz's mean-variance formulation characterizes portfolio risk through the variance of returns and identifies the set of optimal portfolios known as the efficient frontier. Each portfolio on the efficient frontier represents the highest attainable expected return for a specified level of risk. Investors can then select a portfolio that best aligns with their individual risk preferences, thereby achieving an optimal trade-off between risk and return. The capital allocation problem for a portfolio consisting of p assets can be expressed as the following mean-variance optimization model [23]:

$$\begin{aligned} \max_{\mathbf{w}} \quad & \sum_{k=1}^{p} w_k \mu_k - \frac{\rho}{2} \sum_{k=1}^{p} \sum_{l=1}^{p} w_k w_l \sigma_{kl} \\ \text{s.t.} \quad & \sum_{k=1}^{p} w_k = 1 \end{aligned} \tag{1}$$

where p denotes the number of assets in the portfolio, w_k represents the capital weight allocated to the k-th asset, μ_k denotes the expected return of the k-th asset, and σ_{kl} represents the covariance between the returns of the k-th and l-th assets. The parameter $\rho > 0$ reflects the investor's degree of risk aversion, with higher values corresponding to stronger preferences for risk minimization.

Equations (1) can be equivalently reformulated in matrix notation as the following constrained quadratic optimization problem:

$$\begin{aligned} \min_{\mathbf{w}} \quad & \frac{\rho}{2} \mathbf{w}^T \mathbf{\Sigma} \mathbf{w} - \mu^T \mathbf{w} \\ \text{s.t.} \quad & \mathbf{e}^T \mathbf{w} = 1 \end{aligned} \tag{2}$$

where μ is the vector of expected asset returns, w is the vector of portfolio weights, Σ is the covariance matrix of asset returns, and e is a vector of ones with dimension p, defined as

$$\mathbf{e}^T = \begin{bmatrix} 1 & 1 & \cdots & 1 \end{bmatrix}. \tag{2}$$

The objective function in equation (2) is quadratic in the portfolio weights due to the covariance term $w^\top \Sigma w$, representing portfolio risk, while the linear term $\mu^\top \mathbf{w}$ captures expected return.

As a result, the Markowitz portfolio optimization problem belongs to the class of quadratic programming problems. While this formulation is mathematically elegant and widely used, it typically allows for negative portfolio weights, which correspond to short selling. Such solutions motivate the need for alternative optimization approaches when additional constraints such as those imposed by Shariah principles are introduced, as discussed in the subsequent sections.

2.3. Islamic Stock Weighting Constraint

According to Bapepam-LK (Capital Market and Financial Institutions Supervisory Agency) in Regulation Number V.D.6 of 2008, short selling is defined as a securities sale transaction in which the securities are not owned by the seller at the time of the transaction. Investors borrow securities from other parties such as brokers and then sell them in the market. When the price of the security falls, the investor buys back the security to return it to the borrower. The profit from short selling is derived from the difference between the selling price and the purchase price of the security.

The existence of a short-selling system in the stock portfolio directly affects the value of capital weight where there is a negative capital weight value, which means that there is capital from parties other than investors in the form of loans. Consequently, the opposite condition without short selling will create a constraint function that requires all portfolio capital weight values to be non-negative as in the following equation:

$$w_k \geq 0, \quad \forall k,\ k = 1, 2, \ldots, p \tag{5}$$

The constraints are linear equations that can be optimized by quadratic programming. This research uses the Frank-Wolfe method to find the minimal solution of the objective function with both constraint functions. By incorporating the non-negativity constraint into the Markowitz mean-variance formulation, the Shariah-compliant portfolio optimization problem can be written as the following quadratic programming model:

$$\begin{aligned} \min_{\mathbf{w}} \quad & \frac{\rho}{2}\mathbf{w}^T \boldsymbol{\Sigma} \mathbf{w} - \mu^T \mathbf{w} \\ \text{s.t.} \quad & \mathbf{e}^T \mathbf{w} = 1, \\ & \mathbf{w} \geq \mathbf{0}. \end{aligned} \tag{7}$$

This formulation ensures that all portfolio weights remain non-negative, thereby eliminating short selling and maintaining compliance with Shariah investment principles. The feasible region defined by the constraints corresponds to the unit simplex in $\mathbb{R}^p$, which is a convex set suitable for optimization using the Frank-Wolfe algorithm.

2.4. Frank-Wolfe Algorithm

The Frank-Wolfe algorithm is an iterative method for solving constrained convex optimization problems with linear constraints. In this study, the algorithm is applied to solve the convex portfolio optimization problem defined in equations (3), (4), and (6). The general optimization problem can be written as [24]:

$$\begin{aligned} \min_{\mathbf{x}} \quad & f(\mathbf{x}) \\ \text{s.t.} \quad & A\mathbf{x} = \mathbf{b}, \\ & \mathbf{x} \geq \mathbf{0}. \end{aligned} \tag{8}$$

The core idea of the Frank-Wolfe algorithm is to approximate the convex objective function by a linear function at a feasible point, allowing the use of linear optimization techniques. Let

$$\mathbf{x}' = \left[x'_1, x'_2, \ldots, x'_p\right]^T \tag{3}$$

denote an initial feasible solution satisfying the constraints in (8). The objective function $f(x)$ is approximated using a Taylor series expansion around $x = x'$. In general, the Taylor expansion can be written as [25]:

$$f(\mathbf{x}) \approx f(\mathbf{x}') + \sum_{i=1}^{n} \sum_{k=1}^{p} \frac{\partial^i f(\mathbf{x}')}{\partial x_k^i \, i!} (x_k - x'_k)^i \tag{9}$$

In the Frank-Wolfe algorithm, only the first-order term of the Taylor expansion is retained ($n = 1$), since $f(\mathbf{x})$ is assumed to be continuously differentiable and convex. Consequently, higher-order terms are omitted, yielding the following first-order approximation:

$$\begin{aligned} f(\mathbf{x}) &\approx f(\mathbf{x}') + \sum_{k=1}^{p} \frac{\partial f(\mathbf{x}')}{\partial x_k} (x_k - x'_k) \\ &= f(\mathbf{x}') + \nabla f(\mathbf{x}')^\top (\mathbf{x} - \mathbf{x}'), \qquad n = 1. \end{aligned} \tag{10}$$

where $\nabla f(\mathbf{x}')$ denotes the gradient of f evaluated at $\mathbf{x}'$. Since $f(\mathbf{x}')$ and $\nabla f(\mathbf{x}')^\top \mathbf{x}'$ are constants with respect to $\mathbf{x}$, minimizing the first-order approximation in (10) is equivalent to minimizing the following linear objective function:

$$g(\mathbf{x}) = \nabla f(\mathbf{x}')^\top \mathbf{x} \tag{11}$$

or equivalently,

$$g(\mathbf{x}) = \sum_{k=1}^{p} \frac{\partial f(\mathbf{x}')}{\partial x_k} x_k. \tag{12}$$

The resulting linear programming subproblem is then defined as:

$$\begin{aligned} \min_{\mathbf{x}_{LP}} \quad & g(\mathbf{x}_{LP}) \\ \text{s.t.} \quad & A\mathbf{x}_{LP} = \mathbf{b}, \\ & \mathbf{x}_{LP} \geq \mathbf{0}. \end{aligned} \tag{13}$$

The solution $\mathbf{x}_{LP}$ represents a feasible descent direction. However, because the original objective function $f(x)$ is not necessarily monotone along the direction from $\mathbf{x}'$ to $\mathbf{x}_{LP}$, the point $\mathbf{x}_{LP}$ is not directly chosen as the next iterate. Instead, the next trial solution is obtained by performing a line search along the segment connecting $\mathbf{x}'$ and $\mathbf{x}_{LP}$ [26]:

$$\mathbf{x} = \mathbf{x}' + t(\mathbf{x}_{LP} - \mathbf{x}') \tag{14}$$

where the step size $t \in [0, 1]$ is determined by solving:

$$\min_t f\left(\mathbf{x}' + t(\mathbf{x}_{LP} - \mathbf{x}')\right), \quad t \in [0, 1]. \tag{15}$$

The resulting solution is then used as the new feasible point for the next iteration. This process is repeated until convergence criteria are satisfied, such as negligible improvement in the objective function value or attainment of a predefined tolerance level.

The Frank–Wolfe algorithm is initialized with a feasible starting point located at the simplex vertex corresponding to the asset with the highest expected return. The stopping criterion is based on the duality gap, with tolerance set to 10^{-6}. A maximum of 500 iterations is imposed to ensure computational stability. Sensitivity tests indicate that alternative feasible initializations (uniform allocation or random simplex points) produce numerically similar solutions, confirming robustness to initialization.

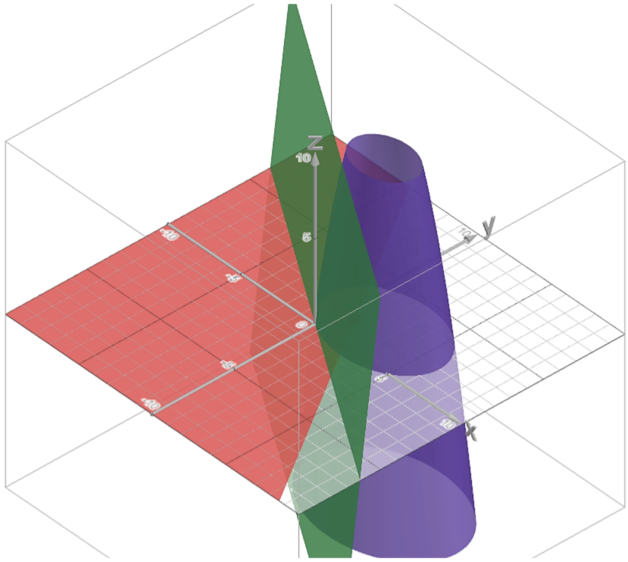

Figure 3. The first iteration of the Taylor series linear approximation.

2.5. *Sharia-Compliant Portfolio Optimization Using Frank-Wolfe Algorithm*

The application of the Frank-Wolfe algorithm to the Islamic portfolio model is appropriate. The objective function of the Markowitz portfolio model needs to be linearly approximated using Taylor expansion. Additional constraints on the Islamic portfolio model in the form of linear inequality can be solved by minimizing the results of linear expansion using the Frank-Wolfe algorithm.

$$g(\mathbf{w}) = \nabla f(\mathbf{w}')^\top \mathbf{w} \tag{16}$$

$$\nabla f(\mathbf{w}') = \rho \Sigma \mathbf{w}' - \mu \tag{17}$$

The vector μ and the constant ρ are constant and have no direct effect on the weight $\mathbf{w}'$ for each variable, so both can be ignored in the derivation process to streamline the objective function. This research uses risk aversion to optimize the model based on the level of risk taken. Equation (16) is the approximated linear objective function obtained:

$$g(\mathbf{w}) = \rho(\mathbf{w}')^T \Sigma \mathbf{w} - \mu^T \mathbf{w} \tag{18}$$

$$\begin{aligned} \min_{\mathbf{w}_{LP}} \quad & g(\mathbf{w}_{LP}) \\ \text{s.t.} \quad & \mathbf{e}^T \mathbf{w}_{LP} = 1, \\ & \mathbf{w}_{LP} \geq \mathbf{0}. \end{aligned} \tag{19}$$

The value of $\mathbf{w}'$ used as the initiation or feasible solution can be determined randomly as long as it satisfies the constraint function used, such as the vector $\mathbf{w}'^T = [1, 0, 0]$. The feasible solution is used to determine the next iteration solution by minimizing the objective function $g(\mathbf{w})$ using equation (18). But the equation $f(\mathbf{w})$ does not guarantee that it is a monotonically decreasing function on the interval of the feasible solution domain and the next solution. Hence the next solution is determined by the optimal point search method on the interval of both using equation (19):

$$\mathbf{w}_{new} = \mathbf{w}' + t(\mathbf{w}_{LP} - \mathbf{w}') \tag{20}$$

$$\min_{t} f(\mathbf{w}' + t(\mathbf{w}_{LP} - \mathbf{w}')), \quad t \in [0,1] \tag{21}$$

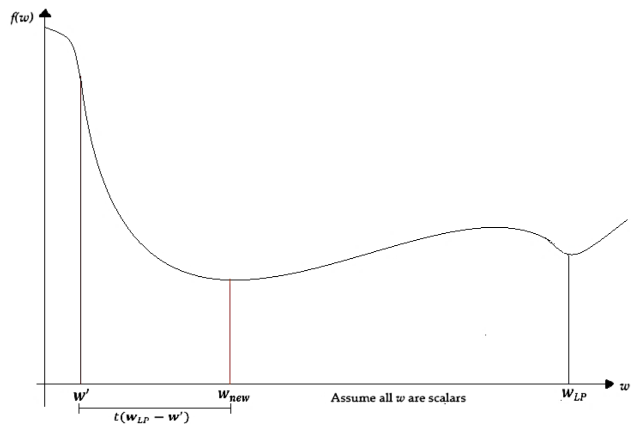

Figure 4. Illustration of Equation (19) optimization.

In such objective function minimization problems, the algorithm iteration process can be stopped using the norm between the value of $\mathbf{w}$ at the i-th iteration and $(i-1)$ because the value of $\mathbf{w}$ is a vector. The norm calculation is performed using

$$\delta^{(i)} = \mathbf{w}^{(i)} - \mathbf{w}^{(i-1)} \tag{22}$$

$$\mathbf{w}^{(i)} = \mathbf{w}^{(i-1)} + t\left(\mathbf{w}_{LP}^{(i-1)} - \mathbf{w}^{(i-1)}\right), \quad \text{if } \|\delta^{(i)}\| > \varepsilon. \tag{23}$$

3. Results and Discussion

3.1. Performance of Sharia-Compliant Portfolio

The out-of-sample performance of the optimized Sharia-compliant portfolios constructed from Jakarta Islamic Index (JII) stocks is evaluated using cumulative wealth, rolling volatility, and rolling Sharpe ratio during the testing period from 2023 to 2024. The portfolios are constructed with varying numbers of assets ($n = 5, 10, 15,$ and 20) and compared with the Indonesian composite stock index (JKSE) as the market benchmark.

3.1.1. Cumulative Wealth Performance Figure 5 illustrates the cumulative wealth evolution of the Sharia-compliant portfolios and the JKSE index. The results show that portfolio performance varies with portfolio size. Smaller portfolios tend to exhibit larger fluctuations in cumulative wealth, reflecting higher exposure to individual asset movements. In contrast, portfolios with more assets display smoother trajectories, indicating improved diversification effects.

While certain Sharia-compliant portfolios outperform the market index during specific subperiods, the JKSE index demonstrates relatively stable growth throughout the evaluation horizon. The optimized Sharia portfolios remain competitive with the market benchmark, particularly when diversification is sufficient. These findings suggest that Sharia-compliant portfolios can provide reasonable growth potential despite operating under long-only and Sharia screening constraints.

3.1.2. Rolling Volatility Analysis The rolling volatility comparison in Figure 6 highlights the risk dynamics of the portfolios. Concentrated portfolios consistently exhibit higher volatility levels, especially during periods of market turbulence. As the number of assets increases, portfolio volatility decreases and becomes more stable over time.

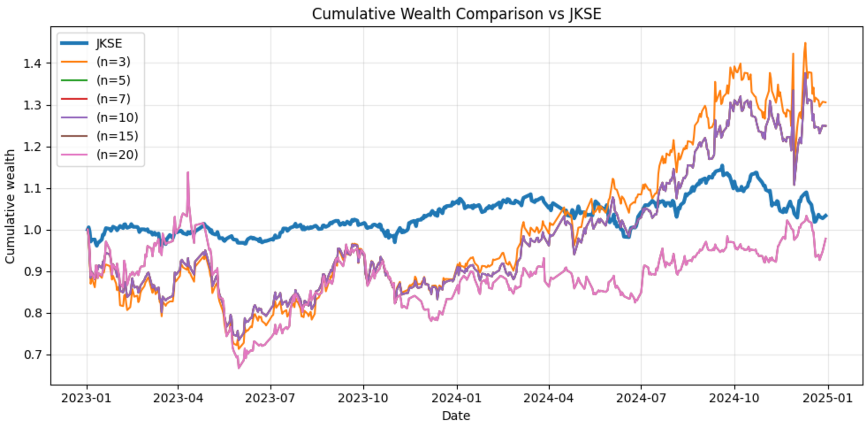

Figure 5. Cumulative wealth comparison.

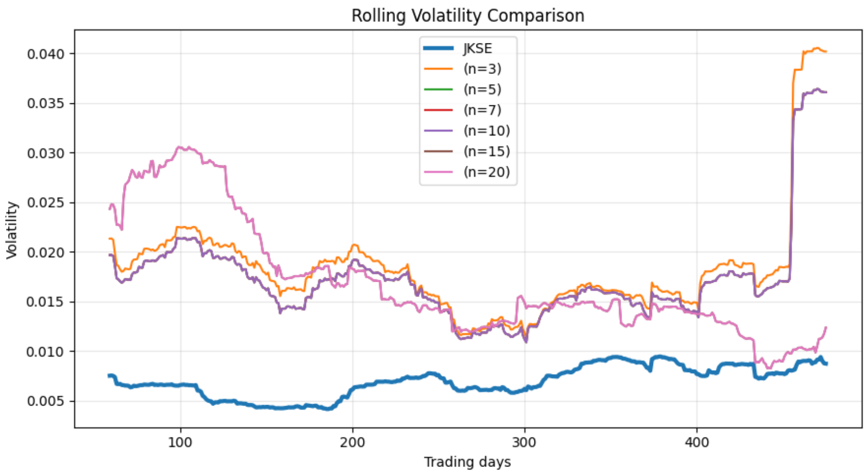

Figure 6. Rolling volatility comparison.

Compared to the market index, the Sharia-compliant portfolios generally show higher short-term volatility for small portfolio sizes. However, this difference diminishes as portfolio size increases, indicating that diversification within the Sharia-compliant universe plays a crucial role in risk reduction. This result aligns with classical portfolio theory, where diversification mitigates idiosyncratic risk.

Table 1. Performance Summary

Portfolio	Mean Return	Volatility	Sharpe Ratio	Max Drawdown
JKSE	0.000069	0.007251	0.009538	-0.117408
n=3	0.000550	0.021909	0.025126	-0.286636
n=5	0.000458	0.020175	0.022711	-0.265622
n=7	0.000458	0.020175	0.022711	-0.265622
n=10	0.000458	0.020175	0.022711	-0.265622
n=15	-0.000016	0.018277	-0.000888	-0.413315
n=20	-0.000016	0.018277	-0.000888	-0.413315

Table 1 reports the out-of-sample performance statistics of the optimized Sharia-compliant portfolios and the market benchmark. The results reveal a clear trade-off between return and risk across different portfolio sizes. Smaller portfolios, such as the $n = 3$ configuration, achieve the highest mean return and Sharpe ratio among the optimized portfolios, but at the cost of substantially higher volatility and deeper maximum drawdowns. This behavior reflects the limited diversification available in highly concentrated portfolios.

As the number of assets increases, portfolio volatility generally decreases, indicating improved diversification within the Sharia-compliant asset universe. However, the reduction in volatility is accompanied by lower average returns, resulting in more moderate Sharpe ratios for larger portfolios.

Compared with the market benchmark, the JKSE index exhibits the lowest volatility and relatively small drawdowns, reflecting its broad market diversification. Nevertheless, several optimized Sharia portfolios achieve higher average returns and competitive Sharpe ratios, suggesting that Sharia-compliant portfolio optimization can provide meaningful risk-return trade-offs despite operating under investment constraints.

3.1.3. Rolling Sharpe Ratio Figure 7 presents the rolling Sharpe ratio of the portfolios and the market index. The results indicate that several Sharia-compliant portfolios achieve risk-adjusted performance comparable to the JKSE index during multiple periods in the testing horizon. Medium-sized portfolios (e.g., $n = 10$ and $n = 15$) demonstrate relatively stable positive Sharpe ratios compared to smaller portfolios.

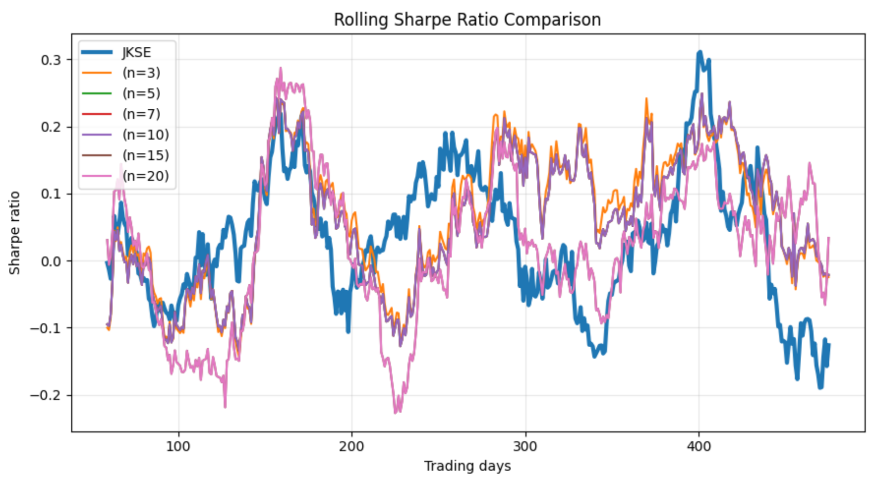

Figure 7. Rolling Sharpe comparison.

In contrast, highly concentrated portfolios show more volatile Sharpe ratio behavior, including extended periods of negative performance. This observation reinforces the importance of portfolio size in maintaining stable risk-adjusted returns under Sharia investment constraints.

Overall, the empirical results indicate that optimized Sharia-compliant portfolios constructed from JII stocks are capable of delivering competitive performance relative to the market benchmark. Although the portfolios do not consistently outperform the JKSE index, they maintain acceptable levels of return while managing risk through diversification.

3.2. Diversification Effects

This subsection investigates the impact of portfolio size on diversification and portfolio stability within the Sharia-compliant investment universe. The analysis considers portfolios constructed from different numbers of JII stocks ($n = 3, 5, 7, 10, 15$, and 20).

3.2.1. Portfolio Risk and Diversification Figure 8 shows the relationship between portfolio volatility and the number of assets. A general downward trend in volatility can be observed as the number of assets increases.

The most notable reduction occurs when moving from small portfolios to medium-sized portfolios, indicating that diversification plays an important role in reducing portfolio risk.

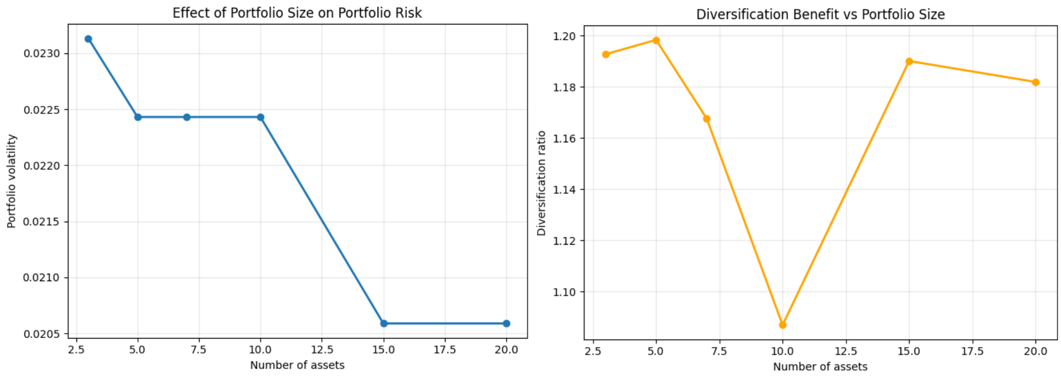

Figure 8. Effect of portfolio size on risk and diversification.

This observation is consistent with the diversification principle in modern portfolio theory, where increasing the number of assets reduces idiosyncratic risk. Although the reduction in volatility becomes less pronounced beyond a certain portfolio size, larger portfolios still exhibit more stable risk profiles than highly concentrated portfolios.

Table 2. Diversification Table

Number of Assets	Expected Return	Portfolio Volatility	Diversification Ratio
3	0.000837	0.023133	1.192684
5	0.000938	0.022431	1.198299
7	0.000938	0.022431	1.167563
10	0.000938	0.022431	1.086889
15	0.001027	0.020589	1.190051
20	0.001027	0.020589	1.181858

Table 2 further quantifies this effect using the diversification ratio. The results indicate that portfolios with more assets generally achieve higher diversification benefits compared to smaller portfolios. In particular, portfolios with 15 and 20 assets display lower volatility while maintaining comparable expected returns, suggesting a more efficient balance between risk and return.

3.2.2. Performance Across Portfolio Sizes Figure 9 presents the cumulative wealth trajectories of portfolios constructed with different numbers of assets during the out-of-sample period. Smaller portfolios demonstrate larger fluctuations and more pronounced drawdowns, reflecting their sensitivity to individual asset movements.

In contrast, portfolios with larger asset sets exhibit smoother cumulative wealth paths and improved stability over time. Although the growth rates across portfolio sizes are relatively similar, larger portfolios show more consistent performance, which reinforces the importance of diversification within the Sharia-compliant stock universe.

Interestingly, the portfolios with 15 and 20 assets achieve relatively stable growth with lower volatility compared to smaller portfolios, indicating that diversification can be achieved even within the limited set of Sharia-compliant equities.

Portfolio size plays a crucial role in stabilizing performance in Sharia-compliant portfolio construction. Increasing the number of assets reduces volatility and improves diversification, although the marginal benefit of diversification diminishes as the portfolio becomes larger.

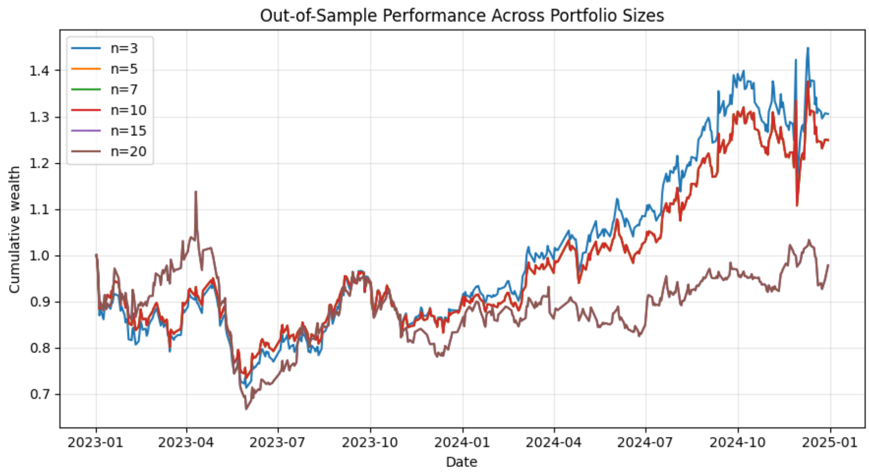

Figure 9. Performance across portfolio sizes.

3.3. Portfolio Allocation under Risk Aversion

This subsection examines how investor risk aversion influences portfolio composition and the resulting risk-return trade-off within the Sharia-compliant investment universe. The analysis is conducted using the portfolio constructed from 20 JII stocks while varying the risk-aversion parameter (ρ).

Table 3. Risk Aversion Table

ρ	Expected Return	Volatility
0.1	0.001067	0.021891
1.0	0.001027	0.020589
1.3	0.000971	0.019353
1.7	0.000910	0.018272
2.0	0.000820	0.016886
2.5	0.000715	0.015429
3.0	0.000641	0.014518
4.0	0.000539	0.013461
5.0	0.000474	0.012901
7.0	0.000398	0.012390
10.0	0.000341	0.012108

Table 3 summarizes the expected return and volatility of the optimized portfolios under different levels of risk aversion. As the risk-aversion parameter increases, portfolio volatility decreases consistently, accompanied by a gradual reduction in expected return. This pattern reflects the fundamental trade-off between return and risk in portfolio optimization.

Figure 10 illustrates this relationship in the risk-return space. The portfolio solutions form a smooth curve resembling the efficient frontier, where lower values of ρ correspond to more aggressive portfolios with higher expected returns and volatility, while higher values of ρ produce more conservative portfolios with reduced risk exposure. The results confirm that the optimization framework allows investors to systematically adjust their portfolio risk profile according to their risk preferences, even within the constraints of Sharia-compliant investment rules.

Figure 11 presents the evolution of portfolio weights as the risk-aversion parameter changes. The allocation patterns reveal clear shifts in asset concentration across different risk preferences. For small values of ρ, the

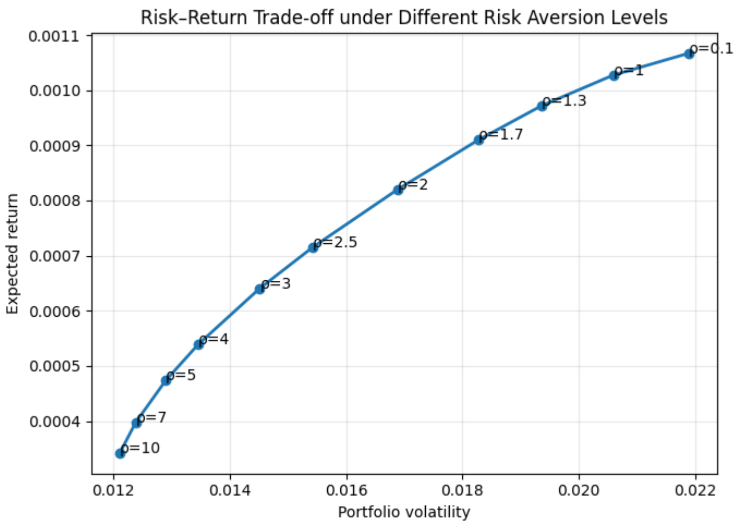

Figure 10. Risk-return trade-off under different risk aversion levels.

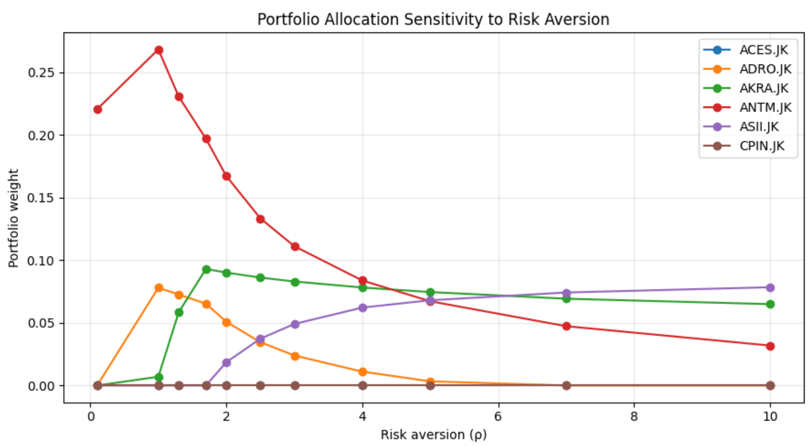

Figure 11. Portfolio allocation by risk aversion levels.

portfolio becomes more concentrated in a few high-return assets, resulting in larger individual asset weights. As risk aversion increases, the allocation becomes more balanced across assets, reflecting a stronger emphasis on risk reduction and diversification.

The selected values of the risk-aversion parameter ρ span from aggressive to conservative investor profiles. Lower values (e.g., $\rho = 0.1$) represent return-seeking investors willing to tolerate higher volatility, while higher values (e.g., $\rho \geq 5$) correspond to risk-averse investors prioritizing capital preservation. Extremely high values approach minimum-variance behavior, while moderate values ($\rho \in [1, 3]$) align more closely with typical institutional allocation preferences.

Several assets maintain relatively stable weights across different risk-aversion levels, indicating their role as core components in the optimized Sharia-compliant portfolio. In contrast, other assets exhibit declining weights as risk aversion increases, suggesting that they contribute more to return than to risk stabilization. These allocation dynamics demonstrate that the optimization model captures economically meaningful portfolio adjustments in response to investor preferences.

The results indicate that the risk-aversion parameter plays a crucial role in shaping both the risk profile and composition of Sharia-compliant portfolios. Lower risk aversion leads to more concentrated portfolios with higher return potential, while higher risk aversion produces more diversified and stable allocations.

3.4. Frank-Wolfe Algorithm Performance

This subsection evaluates the computational behavior of the Frank-Wolfe (FW) algorithm when applied to Sharia-compliant portfolio optimization and compares its solution with the quadratic programming (QP) benchmark.

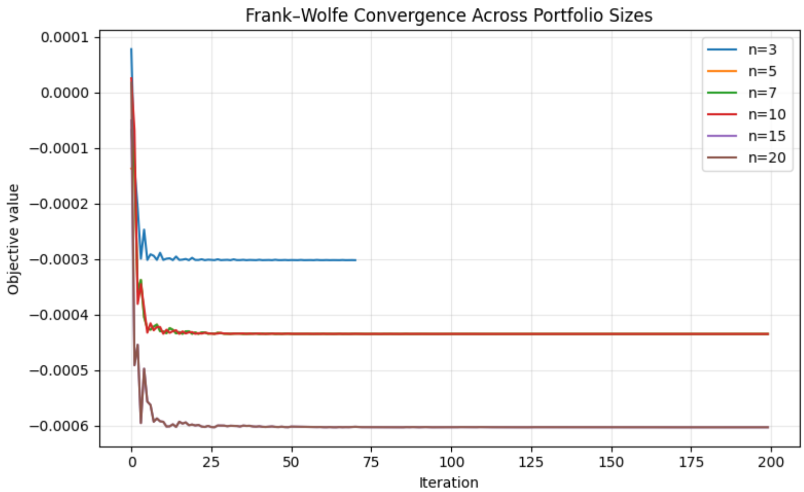

Figure 12. Frank-Wolfe convergence across portfolio size.

Figure 12 illustrates the convergence of the Frank-Wolfe algorithm across different portfolio sizes. The objective value decreases rapidly during the early iterations and stabilizes afterward, indicating reliable convergence behavior. For all tested portfolio sizes, the algorithm reaches a stable objective value within a relatively small number of iterations.

Although larger portfolios require slightly more iterations to stabilize, the convergence pattern remains consistent across all configurations. This behavior confirms that the Frank-Wolfe algorithm can efficiently solve the constrained portfolio optimization problem without requiring projection steps. The smooth convergence pattern also indicates that the quadratic objective function and simplex constraint set form a well-conditioned optimization problem for the Frank-Wolfe method.

Table 4. Frank-Wolfe vs Quadratic Programming Solver Comparison

Assets	FW Objective	QP Objective	Weight Difference Norm
3	-0.000302	-0.000302	0.000502
5	-0.000435	-0.000435	0.003476
7	-0.000435	-0.000435	0.003476
10	-0.000435	-0.000435	0.005601
15	-0.000603	-0.000603	0.013415
20	-0.000603	-0.000603	0.013415

Table 4 compares the optimization results obtained using Frank-Wolfe and the QP solver. The objective values produced by both methods are nearly identical across all portfolio sizes, demonstrating strong agreement between the two optimization approaches.

The norm of the difference between the portfolio weights obtained from FW and QP remains small for all configurations, indicating that the Frank-Wolfe algorithm produces solutions that are numerically close to the exact

QP solution. Slight differences in weights are expected due to the iterative nature of the Frank-Wolfe algorithm and the stopping tolerance used in the implementation. These results confirm that Frank-Wolfe provides an accurate approximation of the optimal solution while maintaining computational simplicity.

The Frank-Wolfe algorithm is capable of solving Sharia-compliant portfolio optimization problems efficiently and reliably. The algorithm converges quickly and produces portfolio allocations that closely match those obtained from the quadratic programming benchmark.

4. Conclusion

This study investigates the construction and performance of Sharia-compliant portfolios using stocks from the Jakarta Islamic Index (JII) under a mean-variance optimization framework. The analysis integrates empirical portfolio evaluation with a projection-free optimization approach based on the Frank-Wolfe algorithm.

The empirical results show that optimized Sharia-compliant portfolios are capable of delivering competitive risk-return performance relative to the market benchmark. Although the portfolios do not consistently outperform the composite stock index (JKSE), they demonstrate stable cumulative wealth growth and reasonable risk-adjusted performance, particularly when the number of assets in the portfolio is sufficiently large. These findings confirm that diversification remains effective within the Sharia-compliant investment universe.

The portfolio size analysis reveals that increasing the number of assets reduces volatility and improves diversification benefits, although the marginal impact diminishes for larger portfolios. This result highlights the importance of adequate asset selection when constructing Islamic investment portfolios.

The risk-aversion analysis further shows that investor preferences play a central role in determining portfolio composition. Lower levels of risk aversion lead to more concentrated portfolios with higher expected returns, while higher risk aversion produces more balanced allocations and lower volatility. These results demonstrate that preference-based portfolio optimization can be effectively implemented within Sharia-compliant investment constraints.

From a computational perspective, the Frank-Wolfe algorithm demonstrates stable convergence behavior and produces solutions that closely match those obtained from the quadratic programming benchmark. This confirms that projection-free optimization methods can be applied reliably in constrained portfolio optimization problems without sacrificing solution quality.

The findings of this study indicate that Sharia-compliant portfolio optimization using JII stocks is both feasible and economically meaningful. The integration of diversification strategies, risk-preference modeling, and efficient optimization techniques provides a practical framework for Islamic portfolio management.

Future research may extend this work in several directions. First, larger Sharia-compliant asset universes, such as the Indonesian Sharia Stock Index (ISSI), could be considered to further examine diversification effects. Second, alternative risk measures such as CVaR or downside-risk-based portfolio models may provide additional insights into Islamic portfolio risk management. Third, dynamic portfolio rebalancing strategies and transaction cost considerations could be incorporated to improve practical applicability. Finally, comparative studies involving other projection-free optimization algorithms or stochastic optimization methods may provide deeper understanding of computational efficiency in Islamic portfolio optimization.

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