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Hybrid Modeling of Currency Circulation Volatility: Evidence from the Central Bank of Iraq

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Abstract The circulating currency is an important aspect of the monetary system in Iraq, as it indicates the activity of the economy and the liquidity of the market and is one of the crucial factors used in the formulation of monetary policy. This study forecasts and predicts currency in circulation dynamics based on monthly data between December 2004 and August 2025 with a training set (2004–2021) and a testing set (2022–2025). The series is discovered to be non-stationary in the levels but stationary on first differencing. Diagnostic tests indicate that there is a serious heteroskedasticity (ARCH effect) and non-normal errors, which is why a hybrid ARMA (1,0)–GARCH (1,1) model is estimated with the Generalized Error Distribution (GED). The best orders in each of the candidate model classes (ARMA, GARCH and hybrid models) were chosen in terms of the lowest Bayesian Information Criterion (BIC). All sample forecasting performance was then compared on the basis of the out-of-sample Root Mean Squared Error (RMSE) when the hybrid ARMA (1,0)–GARCH (1,1) model was found to give the lowest RMSE and therefore have better predictive power. Three out-of-sample post-test period forecasts are produced based on the best model to help in making proactive monetary policy. To support reproducible research, the study provides a free, web-based analytical platform that enables researchers to upload their own datasets and receive instant diagnostics, model selection, forecasts, and downloadable visualizations within the same ARMA–GARCH framework. The paper recommends integrating advanced machine learning architectures, such as GRU, LSTM, and their bidirectional variants, with volatility modeling to further enhance forecast precision and inform forward-looking monetary policy.

Keywords currency in circulation, Returns, ARMA, GARCH, Hybridization, FastAPI, Web-based Statistical Application, Python programming language, pythonanywhere

AMS 2010 subject classifications, 60E05, 62E15

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1. Introduction

Monetary policy instruments are crucial in enabling the central bank to protect price stability; while promoting economic growth. The liquidity of cash markets is crucial to assess the effectiveness of the monetary policy as it is the factor that has a significant impact on the stability of the money supply. However, liquidity depends on extrinsic factors, among which the most significant is the amount of money in the market, which, in turn, is adjusted by a group of external factors, such as the inflow of government deposits and reserves with commercial banks. Significant changes in these factors lead to fluctuations in the money supply and interest rates. Predicting currency in circulation presents a major challenge for central banks. Although the central bank has autonomy in issuing currency in terms of volume, design, and denomination, it often struggles to forecast demand, which is largely determined by the non-banking public sect [1, 2, 3]. The currency in circulation (CIC) in Iraq refers to the paper banknotes that are circulating among the masses of people, and those that are stored in the bank vaults and

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the financial institutions. It is a part of the total money supply and the rest is in the demand and savings accounts. Money circulating constitutes one of the sources of financial stability in the country. There are no denominations of coins in use in Iraq and the country uses only paper banknotes. [4] The Iraqi currency is divided into 7 major denominations, that is, the 50,000, 25,000, 10,000, 5,000, 1,000, 500 and 250. The Central Bank of Iraq (CBI) has the duty of controlling the currency by issuing, distributing, processing, reissuing and destroying of banknotes based on the stipulated monetary and security requirements. The monetary base (M0) which consists of the currency issued (currency outside the banking system plus currency held by banks) decreased by 13.9% at the end of 2024 compared to 2023.)Annual Report: Monetary Policy of the Central Bank of Iraq for the Year 2024, www.cbi.iq).

Empirical studies have shown that hybrid volatility models such as ARMA-GARCH and ARIMA-EGARCH models are more effective than conventional models at capturing the dynamic volatility structure of time series information in a variety of different fields: including cryptocurrency price dynamics, environmental variables and even the oil market. These hybrid specifications have been shown to be better predictors, based on the standard information criteria, including the Akaike Information Criterion (AIC) and the Schwarz Information Criterion (SIC) and, more importantly, their greater ability to detect anomalous or irregular patterns in the datasets [5, 6, 7]. The dynamics of physical money that exists outside of formal banking institutions have been studied using an array of different quantitative methods that include: seasonal ARIMA models, extended currency demand specifications, and examining central-bank balance-sheet indicators. The empirical data always shows that currency in circulation is not a constant but very receptive to salient macroeconomic variables such as real gross domestic product, price level inflation and nominal interest rates. More so, the expansion of digital payment systems moderately impacts, reducing, but not eliminating the use of cash; cautionary and increased economic doubt prevails as the core need of tangible money, especially in new and developing economies [8, 9, 10, 11].This paper examines both institutional and behavioral conditions required to realize the internationalization of domestic currencies using an Agent-Based Model (ABM) which is informed by Behavioral Finance. The results highlight the significance of a well-built monetary policy framework, the active involvement of foreign financial institutions, and the nonlinear impact of investor sentiment as key factors of stability and efficiency in the international treasury markets between banks-forming the pillars of the whole International Currency Circulation (ICC) framework development [12]. Two empirical studies shed light on complicated relationships between the money supply (M1) and the most important macroeconomic variables. A unidirectional cause and effect relationship is noticed in the case of Iraq, such that the change in the money supply and exchange rate triggers the level of inflation, which implies that both the expansion of the money supply and loss of value of the currency amplify the inflation pressure together. In another study, which specifically deals with Indonesia, we find the positive relationship involving the foreign currency sale and money supply becoming empirically significant, thus, opposing the traditional theoretical forecasts and thus, highlighting the complex liquidity relationships in the emerging markets [13, 14]. The current study uses the GARCH (1,1) model to approximate the time-varying volatility of LQ45 stock index in Indonesia and integrates it in a Value-at-Risk (VaR) framework to measure risk. The results support the effectiveness of the model to explain volatility clustering to the extent that it shows a peak of 3.21 per cent daily VaR, which suggests the applicability of GARCH model in managing short-term financial risks [15].

Earlier studies have either focused more on the control of inflation (CIC) in foreign markets or have used ARMA-GARCH models of those financial variables that are not directly correlated with CIC. No empirical study, as of yet, has been able to model the volatility of the CIC of Iraq in an ARMA -GARCH framework or even provide an open-source, Python based prediction tool. The current paper fills this gap by undertaking the preliminary ARMA-GARCH analysis of the CIC of Iraq, as well as by providing a practical API that can be used to make real-time forecasts. Monetary aggregates that have been used periodically in the context of Iraqi setting like M1 or M2 have not been analyzed through volatility dynamics, but Currency in Circulation (CIC) as a direct proxy of cash demand has been analyzed. Analysis of the Iraqi CIC data empirically shows the presence of strong heteroskedasticity and thus, makes the traditional techniques that assume a constant variance between variables (such as ARMA or ARIMA specifications) inadequate. Therefore, a much accurate representation of CIC behavior and the improvement of forecasts are obligatory to move towards GARCH-type models (particularly designed to capture time-dependent volatility changes and volatility cluster) to capture the temporal changes in volatility behavior.

1.1. Research problem

The research problem lies in the difficulty of forecasting the currency in circulation in Iraq, due to its fluctuations and instability, in addition to the scarcity of statistical tools that address these fluctuations and continuous changes in time series.

1.2. Research Objective

The main objective of the paper is to analyze the behavior of the currency in circulation in Iraq and constructing a suitable forecasting model based on the ARMA-GARCH model, which guarantees freedom in dealing with time series experiencing high and sudden volatility, to model them in a way that guarantees future forecasting with minimal errors.

1.3. Research Importance

The importance of this research is enhanced by the fact that it analyzes a key monetary policy tool in Iraq, namely the currency in circulation, which is a single variable in a fluctuating time series with constant changes. This discussion will also enable the scholars to understand the dynamics of the currency in circulation variations and match these variations with the economic variables, which include inflation rates, gross domestic products, foreign exchange rates, interest rates, foreign reserves, etc.

1.4. Research Hypothesis

The ARMA-GARCH Model provides an effective methodology in the development and estimation of currency volatility in circulation in Iraq.

1.5. Research Gap

Despite its theoretical importance and wide recognition in the econometric literature, the ARMA-GARCH model has not been practically employed in studies addressing the behavior of the currency in circulation in Iraq, especially those relying on Python.

2. Methodology

has two salient properties of monetary time series: serial dependence and volatility clustering. ARMA models embrace the average dynamics whereas GARCH embraces the changing volatility which is propelled by exogenou economic shocks. Therefore, the ARMA-GARCH model provides a more exact estimation of CIC as compared to t traditional linear models.

2.1. First Model: Autoregressive Moving Average (ARMA)

The ARMA model is equivalent to the ARMA (p, q) model by defining it as an overall modeling model with an autoregressive (AR) component (whose variables are based on the value and the function is (p: Autoregressive order)) and moving average (MA) component (whose variables are based on the error, and the function is (q: Moving average order)). With careful packaging, the ARMA (p, q) numerical model can be regarded as a direct release of the posterior probability:

$$z_t = c + \phi_1 z_{t-1} + \ldots + \ \phi_p z_{t-p} + \varepsilon_t - \theta_1 \ \varepsilon_{t-1} - \ldots - \ \theta_q \ \varepsilon_{t-q} \tag{1}$$

Where z_t is the time series, c is the constant, ϕ_p is the coefficient associated with autoregression, $_q$ is the coefficient pertaining to the moving average, and ε_t and denotes the stochastic errors [16, 17].

2.2. *Second Model: Generalized Autoregressive Conditional heteroscedasticity (GARCH)*

Especially in the financial field, where volatility and imbalance are common, the generalized autoregressive conditional heteroskedasticity (GARCH) model has become an important theoretical tool and application for econometric series. Unfortunately, such series often exhibit significant heteroskedasticity, and the distribution of the corresponding residuals is not constant over time. Even models for some common stocks are unreliable. With a deeper understanding of the behavior of dynamic waves, we have overturned traditional models one by one, creating more modern models that better describe their dynamic behavior. Following the pioneering work of [18] and the further extension of the GARCH model [19], GARCH-type models not only provide greater flexibility in the modeling of traditional financial time-varying models but also enable better prediction of time-varying volatility. By further exploring the numerical results of the GARCH (p, q) model, it can be converted into a probabilistic representation of the future [20, 21, 22].
The GARCH (p, q) model can be expressed as:'

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^{q} \alpha_i a_{t-i}^2 + \sum_{i=1}^{p} \beta_i \sigma_{t-i}^2 \ , \qquad a_t = \sigma_t \varepsilon_t \tag{2}$$

where σ_t^2 represents the conditional variance at time t, a_{t-i}^2 denotes the squared past errors, and q is order of ARCH Model, p is order of GARCH Model, and α_i And β_i are the model parameters, which must be greater than 0 and satisfy the condition $\alpha_i + \beta_i < 1$ to ensure model stability and a finite variance[23, 24, 25].
The most commonly used model, GARCH (1,1), can be written as follows:

$$\sigma_t^2 = \alpha_0 + \alpha_1 a_{t-1}^2 + \beta_1 \ \sigma_{t-1}^2 \ , \ \ \alpha_1, \beta_1 > 0 \ , \ \alpha_1 + \beta_1 < 1 \tag{3}$$

It is also possible to use the GARCH model alone and include a dummy variable in the variance equation to achieve more precise forecasting results [26].
Using the ARMA model in the mean equation of the GARCH model increases the model's accuracy in analyzing and forecasting the currency in circulation with minimal errors [27, 28].

2.3. *Using Returns Variable Instead of the Original Series*

Renderings are often used in fiscal research, especially when using GARCH models, as they measure the change in the same variable over two consecutive periods. The calculation is usually expressed as follows [29].

$$r_t = \log(\frac{y_t}{y_{t-1}}) \tag{4}$$

where y_t is the value of the time series at time t, and y_{t-1} is the value of the time series at time t-1.

2.4. *Comparison Criteria: LogLik, AIC, BIC, RMSE, MAE*

By comparing the different results of these models, we used the analysis of indicators such as autocorrelation (ACF) and partial autocorrelation (PACF) [30], Akaike's Information Criterion (AIC) , Bayesian probability criterion (BIC) [31], and the log-likelihood (LogLik), and root mean square error (RMSE), and mean absolute error (MAE). While there may be trade-off between data sets or estimation techniques, the BIC was given priority in the joint assessment because it has a better penalty for model complexity, and is asymptotically consistent in terms of selection of the true model given correct specification. Besides, RMSE and MAE were also used as they were also depended on in estimation [32, 33].

2.5. *Residual Diagnostic Tests*

Table 1. summarizes key diagnostic tests applied to model residuals to assess specification adequacy. The Jarque–Bera test evaluates the normality of the residuals, the Ljung–Box Q-test detects residual autocorrelation, and Engle's ARCH–LM test identifies the presence of autoregressive conditional heteroskedasticity. In all cases, a p-value ≤ 0.05 leads to rejection of the null hypothesis, indicating a diagnostic issue that may require model refinement.

Table 1. showing Residual Diagnostic Tests

Name of Tests	Null Hypothesis H_o	If p-value	Test Results
Normality Test	Normally Distributed	≤ 0.05	Non-normal
Autocorrelation Test	No Autocorrelation	≤ 0.05	Autocorrelation present
Heteroskedasticity Test	No ARCH effects	≤ 0.05	ARCH effects present

2.6. Forecasting the Currency in Circulation for 44 Months (Jun 2022 – August 2025)

The forecasting process will span 44 months, from Jun 2022 to August 2025. The predicted values will be compared with the actual observations, and the resulting residuals will be illustrated graphically.

2.7. Research Structure (Flowchart)

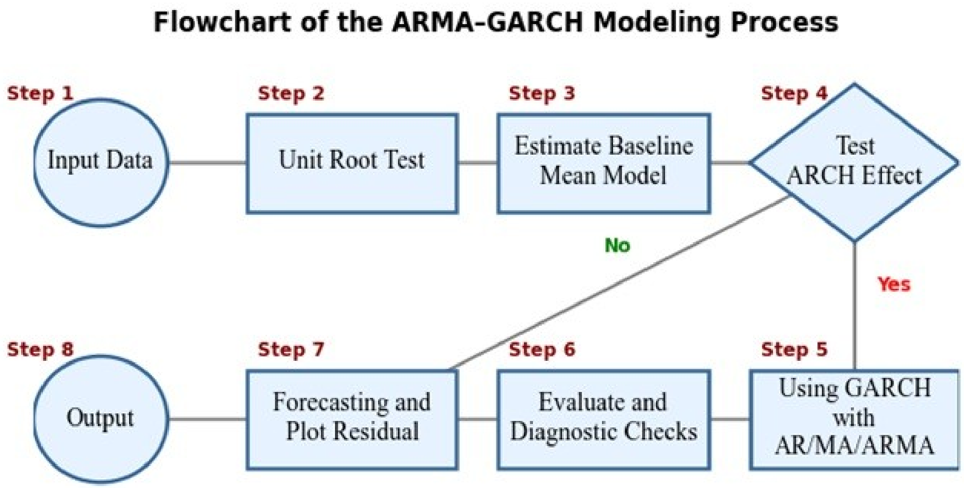

Figure 1. Flowchart of the Modeling Process.

The subsequent flowchart outlines the methodological framework employed to model the logarithmic returns of Currency in Circulation (CIC) using the ARMA–GARCH specification. The process begins with data collection followed by a stationarity assessment using unit root tests, after which a baseline mean model is estimated. Subsequently, Engle's Lagrange Multiplier (LM) test is applied to examine the presence of ARCH effects. If a statistically significant ARCH effect is detected, the optimal orders of both the conditional mean model (AR, MA, or ARMA) and the conditional variance model (GARCH) are selected based on information criteria, specifically the Akaike Information Criterion (AIC) or the Bayesian Information Criterion (BIC), and integrated into a complete ARMA–GARCH specification. Conversely, if no significant ARCH effect is identified, the analysis proceeds with the baseline constant-mean model. The workflow concludes with residual diagnostic checks, out-of-sample forecasting, and the generation of final outputs, including forecasts, diagnostic plots, and performance metrics such as the Root Mean Squared Error (RMSE).

2.8. FastAPI-Based Application Programming Interface (API) for Time Series Analysis

To enhance the practical utility of this research and enable direct access for relevant stakeholders, particularly researchers and central bank staff an Application Programming Interface (API) was developed using the FastAPI framework. FastAPI was selected for its high performance, automatic OpenAPI documentation support, and seamless integration with scientific data processing libraries such as pandas and statsmodels.

https://abdulrazaq1995.pythonanywhere.com/

This tool was specifically designed to analyze and forecast currency issued for circulation in a given country a key monetary indicator often characterized by large numerical values and non-stationary behavior. However, it is equally applicable to any academic researcher analyzing other macroeconomic variables that exhibit similar properties (e.g., GDP, money supply, trade volumes), provided the data consist of positive, large-magnitude time series suitable for logarithmic transformation. The interface allows users to upload two Excel files conforming to specific formatting requirements:

- Data for a single time series (not multiple variables).
- The first file (training data) must contain two columns: `Date` (formatted as `YYYY-MM`) and `Value` (formatted as a numerical value).
- The second file (test data) follows the same structure and is used for out-of-sample model evaluation.

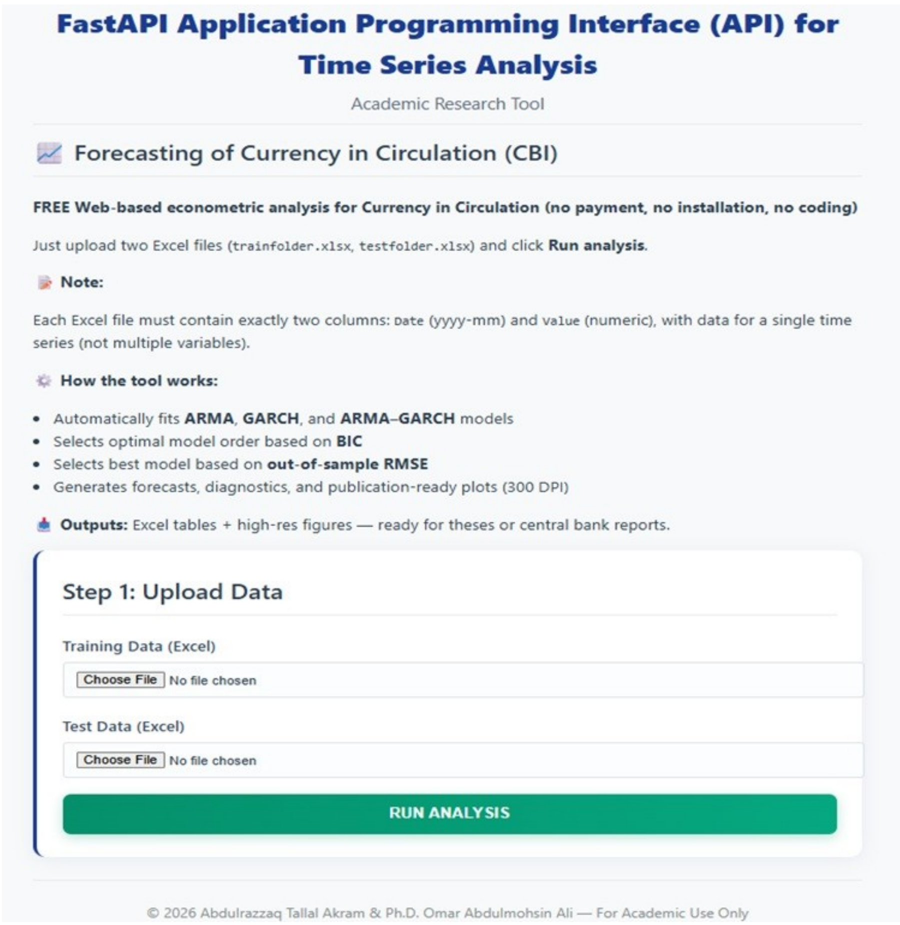

Figure 2. Screenshot of the user interface for the FastAPI-based forecasting tool, showing the file upload section and workflow initiation for currency in circulation analysis.

Once the files are received, the system undertakes an automated and regularized analytical process. Since the target variables (such as the amount of money in circulation) are typically of large magnitude and strictly positive, the system transforms the original series into logarithmic returns using a standard econometric transformation. This preprocessing step is suitable for high-magnitude variables and is commonly employed to stabilize variance, improve distributional properties, and facilitate the efficient application of volatility models such as GARCH and hybrid specifications.

The system then proceeds as follows:

1. Estimates three candidate models (ARMA, GARCH, and ARMA–GARCH) using the log-return series.
2. Selects the optimal model based on the Bayesian Information Criterion (BIC).
3. Generates forecasts for the next three months beyond the test sample using the best-performing model, and subsequently back-transforms the forecasts to the original scale to ensure economic interpretability and practical relevance.
4. Produces a comprehensive Excel report containing:
   - Model parameter estimates.
   - Performance metrics (e.g., RMSE, BIC),
   - Forecasted values expressed in the original data units.
5. Generates five ready-to-download visualizations, including the original time series, observed versus predicted values, model residuals, conditional volatility, and the forward-looking forecast trajectory.

All outputs, including the Excel report and the associated visualizations, are provided via direct download links. This design enables non-technical stakeholders to access computed statistical analyses and publication-ready graphical outputs with minimal effort. Consequently, the proposed system is not only theoretically relevant to the literature on volatility modeling, but also constitutes a practical, deployable instrument for monetary policy formulation and liquidity monitoring, particularly within institutional settings such as the Central Bank of Iraq.

3. Results and Discussion

3.1. Time Series Variable

The dependent variable in this research is the Currency in Circulation in Iraq, which represents the total value of Currency Issued (Outside Banks & Currency with Commercial Banks) by the Central Bank of Iraq for public use. is part of the monetary base as well as a fundamental instrument of monetary policy. The variable is measured in Iraqi Dinars (IQD) [34].

3.2. Data

Based on the monthly data from December 2004 to August 2025, we have a more comprehensive understanding of the dynamic trend of menstrual volume. The sample was divided into two parts: approximately 80% for the training set (2004-2021), and approximately 20% for the testing set (2022-2025). The data were sourced from the official statistical portal of the Central Bank of Iraq (`https://cbiraq.org/`). At the time of writing, this portal is temporarily under maintenance, as confirmed by the relevant officials at the Central Bank. However, updated reports containing historical and current data on currency in circulation remain accessible through the Bank's main website, specifically via the "Key Financial Indicators" section at: `https://cbi.iq/news/view/94`.

3.3. Tools Used in Python programming language

Libraries used: os, warnings, numpy, pandas, matplotlib, datetime, statsmodels, sklearn, arch, library Python as it is able to read in the data.

3.4. Plotting the time series

Figure 3. Shows that the time series of the currency in circulation is non-stationary over time and exhibits a general upward trend.

3.5. Testing the Stationary of the Time Series

Table 2. presents the unit root test on the original data, showing that the series is non-stationary because the p-value is greater than 0.05, leading to the acceptance of the null hypothesis which states that the time series is not stationary.

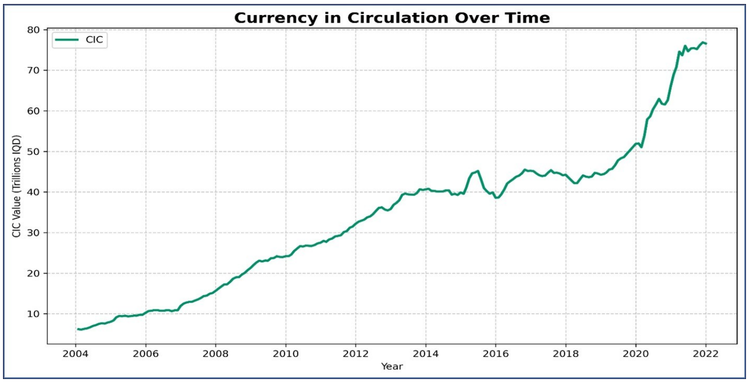

Figure 3. Time series of currency in circulation (CIC) in Iraq.

Table 2. showing the unit root test for the original data

Alternative hypothesis	ADF. Statistic	p. value	Optimal. Lag. BIC
stationary	-1.07	0.9338	2

Table 3. showing the unit root test after taking the first difference of the data

alternative hypothesis	ADF. Statistic	p. value	Optimal. Lag. BIC
stationary	-6.97	0.0000	1

Table 3. presents the unit root test on the data after taking the first difference, showing that the time series is stationary at the first difference because the p-value is less than 0.05. Therefore, the null hypothesis is rejected, and the alternative hypothesis, which states that the time series is stationary is accepted.

3.6. Formulating and drawing returns

$$r = \log(\frac{y_t}{y_{t-1}})$$

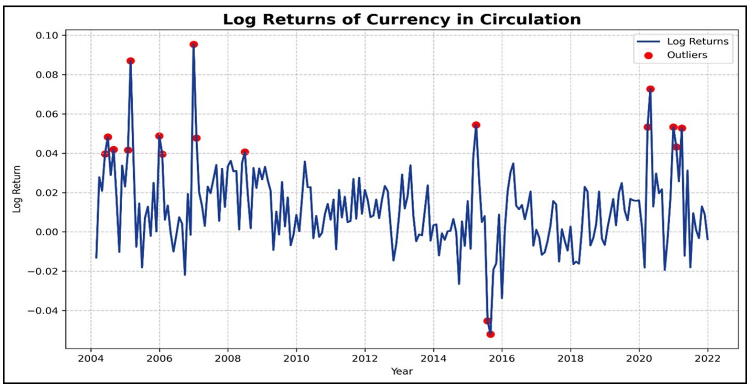

Figure 4. clearly shows the returns and fluctuations of the data.

Figure 4. shows the transformation from the original level series to log-returns reveals pronounced volatility clustering a hallmark of financial time series where periods of high volatility are followed by similar periods, and calm intervals persist over time. This stylized fact justifies the adoption of GARCH-type models to capture the time-varying conditional variance, as standard linear models fail to account for such heteroscedastic dynamics.

3.7. Preliminary Residual Diagnostics

To assess the suitability of classical regression assumptions and guide the choice of an appropriate volatility model, we conduct initial diagnostic tests on the residuals from a simple baseline regression (log returns with constant), examining normality, serial independence, and conditional heteroskedasticity [35].

Table 4. Results of Preliminary Residual Diagnostic Tests

Type of Test	Null hypothesis	Statistic	p-value
Jarque-Bera (Normality)	Normally Distributed	54.52	0.0000
Ljung-Box (Residual ACF(	No Autocorrelation	55.56	0.0000
ARCH-LM (Squared Residuals(	no ARCH effect	10.13	0.0063

Table 4. shows that the residuals from the simple baseline model (log-returns of currency in circulation with a constant) strongly reject normality (Jarque–Bera test, $p < 0.05$), serial independence (Ljung–Box test, $p < 0.05$), and homoskedasticity (ARCH–LM test, $p = 0.0063$). These results reveal that Iraq's monetary system is likely subject to heavy-tailed and asymmetric liquidity shocks, driven by oil revenue volatility, and delayed market adjustments. Such mechanisms generate persistent autocorrelation in cash demand and pronounced volatility clustering in currency circulation, thereby undermining conventional forecasting approaches and justifying the adoption of a hybrid GARCH-type framework that explicitly models time-varying risk and non-linear dynamics for effective liquidity management by the Central Bank [36].

3.8. Comparison Criteria: Autocorrelation (ACF) and partial Autocorrelation (PACF)

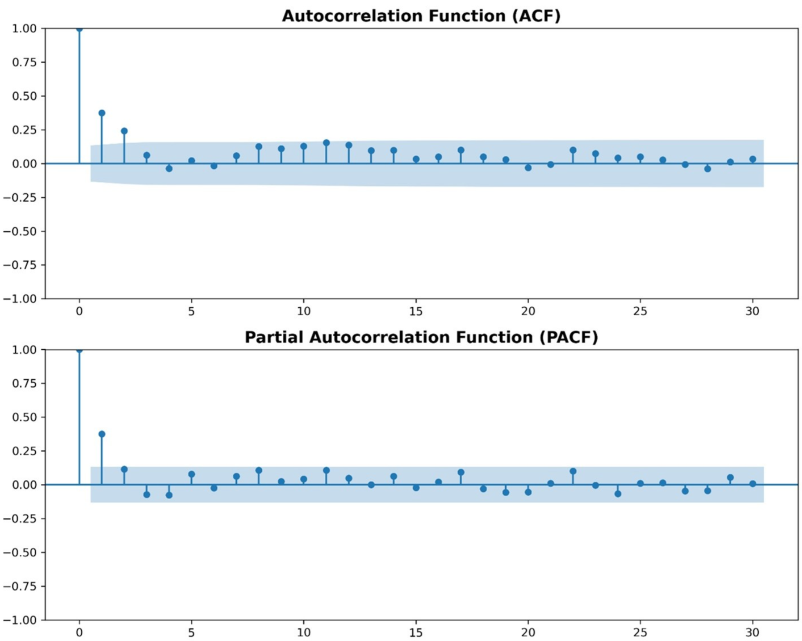

Figure 5. shows the autocorrelation and autocorrelation partial of Log-Returns CIC.

The ACF and PACF of log-returns exhibit significant autocorrelation at lag 1 and a slow decay thereafter, revealing persistent dynamics in Iraq's currency circulation driven by delayed monetary policy transmission, behavioral inertia in cash demand due to underdeveloped financial markets, and prolonged effects of oil-revenue-induced liquidity shocks features that invalidate the assumption of independent errors and necessitate autoregressive modeling prior to GARCH-based volatility estimation.

4. ARMA-GARCH Model Estimation

In this section, the BIC criterion will be adopted as a key metric within each model to select the optimal order, because preliminary diagnostics confirmed the presence of high unobserved heterogeneity in the residuals (ARCH-LM test). Under such conditions, BIC exhibits superior predictive performance compared to AIC/AICc due to its stronger penalty on model complexity, thereby favoring simpler models that avoid overfitting and capture volatility dynamics without excessive sensitivity to sample-specific noise [32].

4.1. Selecting the Best ARMA Model

Table 5. All ARMA (p, q) model results

p	q	BIC	AIC	p	q	BIC	AIC
1	0	-1093.96	-1104.07	0	1	-1085.31	-1095.42
0	2	-1092.44	-1105.92	1	3	-1084.18	-1104.41
2	0	-1091.37	-1104.85	2	2	-1082.84	-1103.06
1	1	-1090.43	-1103.92	3	1	-1082.27	-1102.49
0	3	-1088.44	-1105.3	2	3	-1078.57	-1102.16
1	2	-1088.05	-1104.9	3	2	-1078.22	-1101.81
3	0	-1087.21	-1104.07	3	3	-1073.79	-1100.76
2	1	-1086.44	-1103.3	0	1	-1085.31	-1095.42
Best ARMA model based on BIC							
ARMA (1, 0) with BIC = -1093.96, and LogLik = 555.03							

Table 5. presents the information criteria (BIC and AIC) for all estimated ARMA (p, q) models, with orders restricted to a maximum of 3 for both (p) and (q). This upper bound was chosen based on a substantive economic and methodological rationale: since the data are monthly, lags beyond three months (i.e., one quarter) are deemed implausible in the context of currency issuance and circulation managed by the Central Bank of Iraq. In such a dynamic monetary environment highly sensitive to oil revenues and frequent central bank interventions it is unrealistic to assume that liquidity shocks or policy actions would influence currency demand beyond a quarterly horizon without observable market or institutional responses. Given the presence of high unobserved heterogeneity in the residuals, confirmed by the ARCH-LM test, the BIC criterion was adopted as the primary model selection metric. According to BIC, the optimal model is ARMA (1, 0), which achieves the lowest BIC value and the highest log-likelihood. This indicates that the conditional mean dynamics of log-returns in currency in circulation are adequately captured by a first-order autoregressive process AR (1) without any moving-average component, suggesting that current returns depend primarily on their immediate past value a pattern consistent with short-lived liquidity effects and rapid policy feedback typical of Iraq's cash-demand behavior.

4.2. Selecting the Best GARCH Model

Table 6. All GARCH model results:

GARCH(p,q)	p	q	BIC	AIC	LogLik
GARCH (1,1)	1	1	-1075.037	-1091.890	550.945
GARCH (1,2)	1	2	-1073.683	-1093.907	552.954
GARCH (2,2)	2	2	-1068.313	-1091.907	552.954
GARCH (2,1)	2	1	-1066.487	-1086.710	549.355
GARCH (1,1) with BIC = -1075.04, and LogLik = 550.945					

Table 6. shows BIC, AIC, and log-likelihood values for candidate GARCH (p, q) models. Given the confirmed presence of conditional heteroskedasticity (ARCH-LM) and the monthly frequency of the data, implying that volatility shocks in Iraq's currency market rarely persist beyond two months, the GARCH (1,1) model is selected as optimal based on BIC, as it provides the best balance between fit and parsimony, avoiding overfitting while adequately capturing short-term volatility clustering driven by oil revenues and central bank interventions.

4.3. ARMA - GARCH Model

[7, 16]

Table 7. ARMA (1,0) - GARCH (1,1) Model Results

Dep. Variable:	None		R-squared:	0.14
Mean Model:	AR		Adj. R-squared:	0.136
Vol Model:	GARCH		Log-Likelihood:	563.92
Distribution:	Generalized Error Distribution		AIC:	-1115.84
Method:	Maximum Likelihood		BIC:	-1095.64
No. Observations:	214			
Mean Model				
	coef	std.err	t	p-value
$\widehat{\phi}_0$	0.0063	0.0013	4.886	0.0000
$\widehat{\phi}_1$	0.3945	0.0648	6.089	0.0000
Volatility Model				
$\widehat{\alpha}_0$	0.00002	0.00000001	2693.5	0.0000
$\widehat{\alpha}_1$	0.082	0.07	1.1	0.2710
$\widehat{\beta}_1$	0.86	0.06	13.5	0.0000
Distribution				
	coef	std.err	t	p-value
nu	1.38	0.21	6.7	0.0000

Table 7. presents the estimation results of the hybrid ARMA (1,0)-GARCH (1,1) model. The statistical significance of all the estimated coefficients is high at 1 percent. Estimation of the model indicates that any change in currency in circulation has an immediate effect on the conditional mean through the first-order autoregressive component and the coefficients related to the ARCH (1) and GARCH (1) specifications are the aggregate representation of the persistent but stable volatility dynamics. Particularly, the sum of the GARCH coefficients is 0.923, which is less than one and meets the condition of covariance stationarity. The estimation procedure was chosen as Generalized Error Distribution (GED) since the diagnostic tests especially the Jarque-Bra test rejected the normality of the model assuming that the residuals are normally distributed. The use of GED reduces the chances of biased inference, false predictions or unreliable confidence interval that could have been caused by the false assumption of Gaussian errors, particularly when shock distributions have heavy tails. The parameter of tail-thickness (n=1.38) is an estimated value that qualifies a strong non-normality deviation and enhances predictive power in the strong monetary environment of Iraq.
The model estimation will include two equations. The first is the average equation, which can be written according to the best model chosen, which is ARMA (1,0), as follows:

$$r_t = \phi_0 + \phi_1 r_{t-1} + \varepsilon_t \tag{5}$$

$$r_t = 0.0063 + 0.395\, r_{t-1} \tag{6}$$

The autoregressive coefficient, which equals 0.395, indicates that an increase in currency demand in one period is followed by a partial increase in the next period, reflecting moderate persistence in monetary demand behavior due to factors such as economic uncertainty or slow adjustment to shocks. The second equation is the variance equation and can be written according to the best model chosen, which is GARCH (1,1), as follows:

$$\sigma_t^2 = \alpha_0 + \alpha_1 a_{t-1}^2 + \beta_1\, \sigma_{t-1}^2 \tag{7}$$

$$\sigma_t^2 = 0.00002 + 0.082\, a_{t-1}^2 + 0.86\, \sigma_{t-1}^2 \tag{8}$$

Intercept parameter α_0 represents volatility of the asymptotic baseline in the occurrence of the antecedent shock or historic variations. The relatively small scale shows that there is no major stochastic volatility taking place in the Iraqi economy at times of equilibrium so that fluctuations are mainly caused by exogenous disruptions like crisis or similar shocks. The autoregressive coefficient α_0 measure the short-term reaction of currency-demand volatility to a foregone disturbance during the previous period, e.g. a large-scale cash outflow, a large pay payment, or fluctuation in exchange rates. Since this value is low, it implies that unexpected shocks have only a limited initial effect on volatility possibly due to the slow transmission of shocks through the economy or the presence of preliminary mechanisms to contain and mitigate shocks as much as possible. The persistence coefficient β_1 reflects the degree to which volatility carries over from one period to the next, even in the absence of new shocks. Given its high value, it indicates that the effects of shocks on liquidity volatility persist for extended periods, events such as oil price fluctuations or political crises leave lasting impacts on public confidence and monetary behavior, making liquidity management an ongoing challenge for the Central Bank. An important note is that the sum of the coefficients is less than 1, indicating that the model is stable, however, convergence is slow due to the high value of the β_1 coefficient.

4.4. Comparison Criteria: LogLik, AIC, BIC, RMSE

Subsequent to the identification of the optimal orders for the constituent ARMA and the GARCH models, we move on with the formulation of the hybrid ARMA-GARCH model based on pre-determined specifications. The following table compares the prominent differences between the three modelling paradigms: ARMA, GARCH and ARMA-GARCH Models.

Table 8. Optimal Order Selection for ARMA, GARCH, and Hybrid Models

Model	LogLik	BIC	AIC
ARMA (1,0)–GARCH (1,1)	563.92	-1095.64	-1115.84
ARMA (1,0)	555.03	-1093.96	-1104.07
GARCH (1,1) + Const	550.95	-1075.04	-1091.89

Table 8. presents the in-sample model selection results based on the log-likelihood (LogLik), Akaike Information Criterion (AIC), and Bayesian Information Criterion (BIC). The hybrid ARMA (1,0) – GARCH (1,1) model achieves the highest log-likelihood, and the smallest value AIC and BIC among all candidate models. Since lower (more negative) values of AIC and BIC indicate better trade-off between goodness of fit and model parsimony, these results confirm that the hybrid specification captures the joint dynamics of the conditional mean and time-varying volatility more effectively than either the standalone ARMA (1,0) or GARCH (1,1) + Const. models.

Table 9. In-Sample and Out-of-Sample Forecast Accuracy Comparison

Model	RMSE in Sample	RMSE Out of Sample
ARMA (1,0)–GARCH (1,1)	0.0119	0.0165
GARCH (1,1) + Const	0.0198	0.0168
ARMA (1,0)	0.0183	0.0173

Table 9. presents a comparative assessment of in-sample and out-of-sample forecasting performance across the three candidate models, using the root mean squared error (RMSE) as the evaluation metric. The hybrid ARMA (1,0)–GARCH (1,1) model achieves the lowest RMSE both in-sample (0.0119) and out of sample (0.0165), outperforming the standalone ARMA (1,0) and GARCH (1,1) + Const specifications. Although the differences in out of sample RMSE are modest (e.g., 0.0165 vs. 0.0168 for GARCH), the consistent superiority of the hybrid model underscores the added value of jointly modeling the conditional mean and time varying volatility capturing dynamics that neither component alone can fully represent.

4.5. Residual Diagnostic Tests

Table 10. presents the p-values of three post-estimation diagnostic tests, Jarque-Bera (normality), Ljung-Box (autocorrelation, lag=10), and ARCH-LM (heteroskedasticity) applied to the standardized residuals of the three candidate models. All models strongly reject the null hypothesis of normality ($p = 0.0000$), confirming the presence of heavy tails or skewness, consistent with the non-Gaussian nature of monetary returns in emerging economies

Table 10. The results of Tests

Name of Tests	Normality Test	Autocorrelation Test	Heteroskedasticity Test
ARMA (1,0)–GARCH (1,1)	0.0000	0.939	0.955
GARCH (1,1) + Const	0.0000	0.927	0.918
ARMA (1,0)	0.0000	0.947	0.962

like Iraq. However, all three models, including GARCH (1,1) with a constant mean, exhibit high p-values (¿0.90) for both autocorrelation and heteroskedasticity tests, indicating that none of the models leave significant serial correlation or ARCH effect in the residuals. This suggests that even the simpler specifications adequately capture the conditional mean and variance dynamics. Nevertheless, the hybrid ARMA (1,0)–GARCH (1,1) model remains preferable due to its superior fit (lowest BIC) and forecasting accuracy (lowest out-of-sample RMSE), as shown in Tables (8) and (9). The use of the Generalized Error Distribution (GED) effectively mitigates the impact of non-normality on inference and prediction.

5. Plot the expected value with the actual value

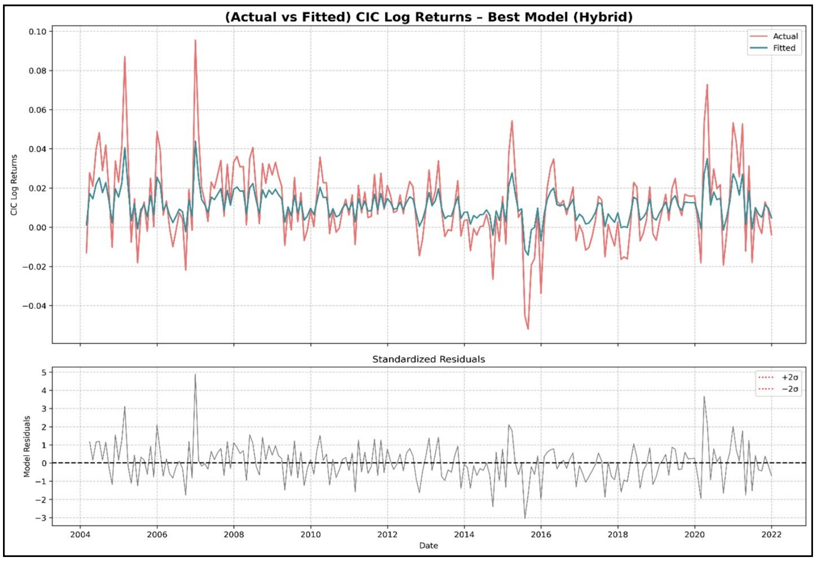

Figure 6. shows the estimation results based on the most appropriate model among the estimated.

Figure 6. Actual versus fitted values of the first-differenced log-return series. The close correspondence between observed and model-implied dynamics indicates that the ARMA (1,0)–GARCH (1,1) specification adequately captures both the conditional mean and time-varying volatility structure of Currency Issued in Iraq. Notably, the model successfully tracks periods of heightened volatility characterized by volatility clustering without exhibiting systematic bias, thereby corroborating its in-sample adequacy and structural validity.

6. forecasting

Figure 7 illustrates the forecasting of the Iraqi currency circulating based on a calculation of the ARMA (1,0)-GARCH (1,1) specification for the period from June 2022 to August 2025. The model was statistically robust as reflected by the 95% prediction intervals and successfully addressed the issues of autocorrelation

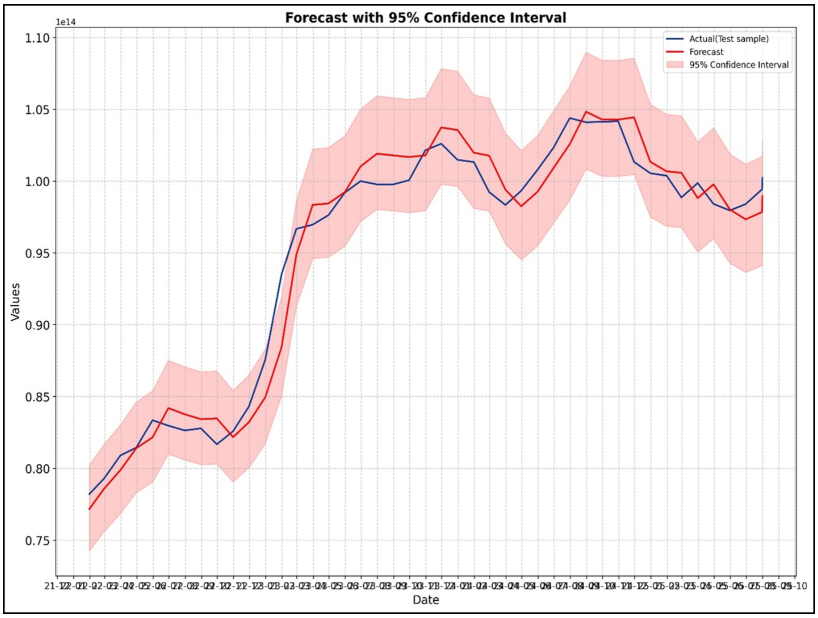

Figure 7. shows the forecasting future values and the variation in them.

and heteroskedasticity. To correct for nonnormality a Generalized Error Distribution (GED) was applied to the residuals, which were confirmed by conventional diagnostic tests.

Table 11. Three Month Ahead Forecasts Beyond the Test Sample

month	September-2025	October-2025	November-2025
value	99,865,099,650,183	99,718,598,619,467	99,636,531,181,443

Table 11. presents the forecasts indicate a slight but persistent decline in currency in circulation over the next three months, from approximately 99.87 trillion Iraqi dinars in September 2025 to 99.64 trillion dinars in November 2025.
This modest downward trend may reflect relatively stable demand for cash liquidity or the initial effects of mildly contractionary monetary policies, such as reduced seasonal government spending or increased public confidence in non-cash alternatives. However, the magnitude of the decline is small (less than 0.25% over three months), suggesting continued relative stagnation in cash circulation without major shocks. This behavior aligns with the stabilized dynamics of the time series after first differencing, as captured by the ARMA (1,0)–GARCH (1,1) model.

7. Conclusions

1. The econometric analysis reveals that the amount of money in circulation in Iraq has time-varying nature, the fact that level series has non-stationarity and stationarity after first differencing transformation supports this observation. This trend is indicative of the inherent interdependence between monetary policy and macroeconomic determinants where in the amount of active money would adjust itself to changes in economic activity, government revenues and fiscal policy in a monetary management structure that pursues price stability.
2. Volatility clustering of the logarithmic returns of the currency in circulation is a common characteristic of an economy which is in structural self-reorganization and institutional modernization. This behavior has highlighted the sensitivity of the cash demand to both external shocks including global market volatility and

domestic shocks including energy prices changes as the Central Bank struggles to keep the transmission of shocks as low as possible in order to hold the liquidity levels normal within the market.

3. The diagnostic results of the first model showed the major violation of the key statistical assumptions)Jarque-Bra statistic provided a strong result of the test of the normality, Ljung-Box test showed the presence of autocorrelation in the residuals, and the ARCH-LM test showed the existence of heteroskedasticity (. To address these shortcomings, three candidate models were estimated, ARMA (1,0), GARCH (1,1), and a hybrid ARMA (1,0) -GARCH (1,1) with Generalized Error Distribution (GED), to model non-normal errors. The post-estimation diagnostics illustrate that all 3 specifications remove autocorrelation and heteroskedasticity (p -values are greater than 0.9 on both Ljung -Box and ARCH -LM tests), but the normality is rejected (p -value is less than 0.05) in all the models, which once again justifies the use of GED. These findings support the notion that the statistical features observed are the features of the currency circulation.
4. Comparison results of models have shown that the hybrid ARMA (1,0)-GARCH (1,1) model is better performing when compared to the independent ARMA and GARCH models in terms of statistical fit and forecasts. It has the lowest Bayesian Information Criterion (BIC) in-sample, which is an indication of optimal tradeoff between model adequacy and parsimony. It achieves the smallest Root Mean Squared Error (RMSE) in that out-of-sample (2022-2025 testing), which legitimizes its better ability to both capture short term persistence in currency demand and time-varying volatility. That is why the incorporation of such hybrid models is consistent with the current best practices at central banks in terms of monitoring and managing monetary liquidity.
5. The conditional variance estimates indicate that the effects of the monetary shocks are highly linked to the larger environment in the economy where the policy is applied, and not to the shortcomings of the instruments used. This observation supports the active position of the Central Bank in dealing with fluctuations in liquidity by use of regulatory and supervisory measures hence maintaining the stability of monetary systems and people confidence on the national currency.
6. Finally, in-sample and out-of-sample predictive assessments indicate that the combination of both the mean and variance dynamics in a single modeling process strengthens the analysis support offered to monetary policy makers. The results reveal the need to constantly improve the quantitative instruments that the Central Bank embraces since they enhance accuracy of liquidity data prediction and efficiency of monetary planning in the constantly changing economic environment.

8. Recommendations

1. The paper suggests that the online platform developed in this study, which is based on FastAPI, should be utilized by the relevant authorities to analyze and predict the currency circulating in Iraq through regular data updates. The platform provides a practical and versatile econometric analysis and short- to medium-term forecasting framework, thereby enabling evidence-based monetary decision-making and enhancing the effectiveness of liquidity management (`https://abdulrazaq1995.pythonanywhere.com`).
2. Taking into consideration the presence of volatility in the relationship of currency circulation, discussed in the context of the lack of the analyzed relationships, it is suggested to introduce indicators of volatility in conventional liquidity evaluation models. These type integrations can help to improve early notification of the liquidity strains and the optimization of monetary policy responses in timing and calibration, especially when the economic uncertainty is high.
3. The paper highlights the importance of continually improving statistical models used in monetary liquidity analysis, and to introduce artificial intelligence tools, in particular the deep neural network models, as auxiliary models to the traditional econometric models. These include both unidirectional and bidirectional systems, including LSTM and GRU, respectively, and BiLSTM and BiGRU, respectively, which have demonstrated strong possibilities of nonlinear patterns and complex time series relationships in monetary time series, improving predictive accuracy and enabling active monetary planning.

4. The research recommends the further institutionalization of the use of modern programming environments such as Python-based analytical environments in the monetary analysis units. These conditions are dynamic, open and repeatable and they are in line with the best practices used by central banks around the world to analyses real time data and make predictions.
5. Considering that currency circulation is sensitive to the macroeconomic environment, it is possible that future analytical models include the important macroeconomic variables, including inflation, GDP, exchange rates, interest rates, and foreign reserves, into extended hybrid or multivariate models. The strategy would enhance the explanatory strength of liquidity analysis without compromising on the proactive and proactive policy of the Central Bank.
6. Lastly, maintaining support on research efforts that explore the relationship between real money circulation and new financial solutions, such as digital payment systems and future Central Bank Digital Currency (CBDC) developments, should be maintained. The result of such research will also be an encompassing understanding of the changing monetary dynamics and the basis of future strategic planning on the future of cash management.

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