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Results on the Grundy Chromatic Number of Prism Graphs

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Abstract A Grundy coloring of a graph G is a proper vertex coloring with positive integers such that for any two colors i and j with $i < j$, every vertex colored j has at least one neighbor colored i. In this paper, we determine the Grundy chromatic number for several important families of graphs, including the prism graph, the n–crossed prism graph, the line graph of the crossed prism graph, and the antiprism graph. Whenever appropriate, illustrative examples are provided to demonstrate the corresponding Grundy colorings.

Keywords Grundy chromatic number, prism graph, crossed prism graph, antiprism graph.

AMS 2010 subject classifications 05C15, 05C38, 05C76, 05C78.

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1. Introduction

Graph theory has evolved into one of the most dynamic and influential areas of modern mathematics. Its origins can be traced back to the seminal work of Leonhard Euler in 1736, in which he resolved the famous Königsberg bridge problem and, in doing so, laid the foundations for an entirely new mathematical discipline. Euler's insight-that the essential features of the problem concerned connectivity rather than geometry-introduced the concept of representing real-world situations through abstract structures composed of vertices and edges. Because of this foundational contribution, Euler is widely regarded as the "Father of Graph Theory." Since that time, graph theory has developed into a rich and far-reaching field, with powerful theoretical frameworks and applications spanning numerous scientific and engineering domains.

A graph, in its simplest form, is a mathematical structure used to model pairwise relationships among a set of objects. These objects, called vertices, may represent people, devices, molecules, or abstract units, while the connections between them, called edges, capture interactions or relationships. Due to its versatility, graph theory plays an essential role across a diverse range of disciplines, including computer science, communication networks, physics, chemistry, biological systems, linguistics, operations research, and various branches of social sciences. Comprehensive surveys of graph-theoretic principles and applications can be found in [1, 4, 5, 9, 13], which highlight both foundational concepts and advanced developments in the study of graph structures, colorings, and algorithmic challenges.

One particularly important branch of graph theory is graph coloring, a topic that continues to attract extensive attention because of its theoretical depth and practical relevance. A proper coloring of a graph is an assignment of positive integers (called colors) to the vertices of the graph such that adjacent vertices always receive distinct

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colors. The classical chromatic number, denoted $\chi(G)$, is the minimum number of colors needed for a proper vertex coloring of a graph G. Graph coloring problems arise naturally in scheduling, frequency assignment, compiler design, clustering, and various allocation problems, making them among the most widely studied topics in discrete mathematics. Early developments on chromatic properties, including structural characterizations and inequalities, appear in the works of Simmons [12] and Jensen and Toft [9], among others.

A refinement of proper coloring, known as *Grundy coloring*, emerged indirectly through ideas introduced by Patrick Michael Grundy (1917-1959) in the context of combinatorial game theory. Although Grundy did not formulate the concept explicitly in graph-theoretic terms, ideas from his study of impartial games inspired definitions that later became essential in greedy colorings and online coloring strategies. A Grundy coloring of a graph G is a proper coloring in which each color class appears "as early as possible" in the sense that every vertex colored with a color j must have neighbors colored with every color i for all integers $i < j$. Equivalently, higher-numbered colors can only occur if all lower-numbered colors are forced by adjacency constraints. Consequently, every Grundy coloring is a *complete* coloring, and its color classes form a nested structure dictated by the adjacency patterns.

The connection between Grundy colorings and greedy algorithms is particularly striking. Consider an ordering

$$\phi : v_1, v_2, \ldots, v_n$$

of the vertices of a graph G. A *greedy coloring* assigns colors one vertex at a time, always choosing for v_t the smallest positive integer that does not appear among its colored neighbors. As shown in [2, 6, 7], every greedy coloring is indeed a Grundy coloring, and different orderings of vertices produce different Grundy colorings. This makes the Grundy number an inherently algorithmic and ordering-dependent parameter: it reflects the maximum number of colors that a greedy algorithm can be "forced" to use on a given graph.

Formally, the *Grundy chromatic number* of a graph G, denoted by $\Gamma(G)$, is the largest integer k for which G admits a Grundy k-coloring. If a graph has a Grundy k-coloring, then every vertex assigned the highest color k must be adjacent to vertices of all other colors $1, 2, \ldots, k-1$. This requirement forces strong structural constraints: in particular, such a vertex must have degree at least $k-1$, which immediately yields the upper bound

$$\Gamma(G) \leq \Delta(G) + 1,$$

where $\Delta(G)$ denotes the maximum degree of the graph. Since Grundy colorings are proper colorings, they also satisfy the lower bound

$$\chi(G) \leq \Gamma(G),$$

placing the Grundy number between the classical chromatic number and one greater than the maximum degree. Investigations into cases where $\Gamma(G) = \chi(G)$, or where the inequality is strict, have been explored in [3, 17]. These works highlight the complexity of analyzing greedy colorings and show that computing the Grundy number is NP-hard for general graphs.

Applications of Grundy colorings arise naturally in areas involving online decision making, scheduling, and resource allocation, where items must be assigned to categories sequentially without knowledge of future events. Research on such topics can be found in [5, 7], where first-fit and online colorings are analyzed through structural and combinatorial techniques. Further studies on Grundy colorings of specific families of graphs—such as trees, hypercubes, and bipartite graphs—appear in [6, 7, 17], providing explicit formulas and bounds for important classes of graphs.

Beyond general coloring theory, several researchers have examined specialized colorings and decomposition problems related to different graph families. Results on equitable colorings of Helm graphs, Gear graphs, coronas of wheels, and various sunlet graph families are found in [10, 11, 16]. Harmonious colorings of double star graphs are studied in [14], illustrating how structural restrictions can dramatically change colorability conditions. Moreover, results related to greedy structures and their recognition appear in [1], establishing foundations that connect greedy algorithms with structural graph parameters.

In recent years there has been increasing interest in applying Grundy colorings to highly structured graphs such as prisms, crossed prisms, antiprisms, and line graphs of these constructions. These graph families are of special interest because they arise in studies of polyhedral structures, network topologies, and communication architectures. For example, prisms and crossed prisms model cylindrical or ladder-like frameworks, while antiprisms provide triangulated structures with rich symmetry. Their line graphs, in turn, capture adjacency relationships among edges, making them useful in scheduling and frequency assignment where conflict graphs are naturally line graphs of underlying communication patterns. Foundational work related to prism graph families and domination properties appears in [8], while coloring properties, particularly equitable and greedy colorings, are developed in [15].

The Grundy chromatic number of these graph families presents interesting challenges. Because greedy colorings are sensitive to vertex ordering, determining $\Gamma(G)$ often requires constructing explicit colorings and proving that no larger coloring is possible. This involves analyzing adjacency patterns, degrees, neighborhood structures, and recursive extensions of coloring sequences. For prism graphs and crossed prism graphs, the interplay between the two cycles and the vertical connecting edges requires careful combinatorial analysis to ensure that every color from 1 to $k-1$ appears in the neighborhood of vertices colored k. For antiprism graphs, the alternating connections and triangulated faces impose additional constraints that affect both achievable color classes and upper bounds. In the case of line graphs of crossed prism graphs, the structural change from vertex-based to edge-based interactions often increases local density, requiring more refined bounding arguments.

The present study focuses on determining the Grundy chromatic number for the prism graph, the n–crossed prism graph, the line graph of the crossed prism graph, and the antiprism graph. Our goal is to establish exact values for $\Gamma(G)$ in each case, supported by constructive colorings and rigorous structural arguments. Illustrations are provided where appropriate to demonstrate explicit coloring patterns and to highlight the combinatorial constraints that govern the Grundy number in each graph family.

By combining greedy procedures, structural decomposition, and detailed case analyses, we derive tight bounds and exact formulas for the Grundy number of these graphs. Since the studied graph families are symmetric, well-structured, and of practical significance in network modeling, the results presented here enrich the set of known exact Grundy numbers and contribute to the broader understanding of greedy and online coloring behaviors in graph theory.

2. Preliminaries

In this section we recall several standard graph constructions that will be used throughout the paper.

Definition 2.1
[8] A *prism graph*, denoted by CL_n (also called the *circular ladder graph*), is the graph obtained from the skeleton of an n-prism. It has $2n$ vertices and $3n$ edges, consisting of two n-cycles joined by a perfect matching.

Definition 2.2
[8] An *antiprism graph*, denoted by Q_n, is the graph corresponding to the skeleton of an n-antiprism. It contains $2n$ vertices and $4n$ edges, formed from two n-cycles with alternating diagonal connections.

Definition 2.3
[8] Let $n \geq 4$ be an even integer. An *n-crossed prism graph*, denoted by R_n, is obtained from two disjoint n-cycles by adding crossing edges so that each vertex u_i of the first cycle is adjacent to either v_{i+1} (for odd i) or v_{i-1} (for even i) in the second cycle. This produces a crossed (or twisted) cylindrical structure.

Definition 2.4
[11] The *line graph* of a graph G, denoted by $L(G)$, is the graph whose vertices correspond to the edges of G. Two vertices of $L(G)$ are adjacent if and only if their corresponding edges in G share a common endpoint.

3. Main Results

Theorem 3.1
For every integer $n \geq 3$, the Grundy chromatic number of the prism graph CL_n satisfies

$$\Gamma(CL_n) = 4.$$

Proof
The prism graph CL_n consists of two n–cycles

$$U = \{u_1, u_2, \ldots, u_n\}, \qquad V = \{v_1, v_2, \ldots, v_n\},$$

together with a perfect matching joining u_i to v_i for all i. Thus,

$$E(CL_n) = \{u_i u_{i+1},\, v_i v_{i+1} : 1 \leq i \leq n-1\} \cup \{u_1 u_n,\, v_1 v_n\} \cup \{u_i v_i : 1 \leq i \leq n\}.$$

Each vertex has degree 3, and therefore

$$\Gamma(CL_n) \leq \Delta(CL_n) + 1 = 4.$$

We now construct explicit Grundy 4-colorings for odd and even n.

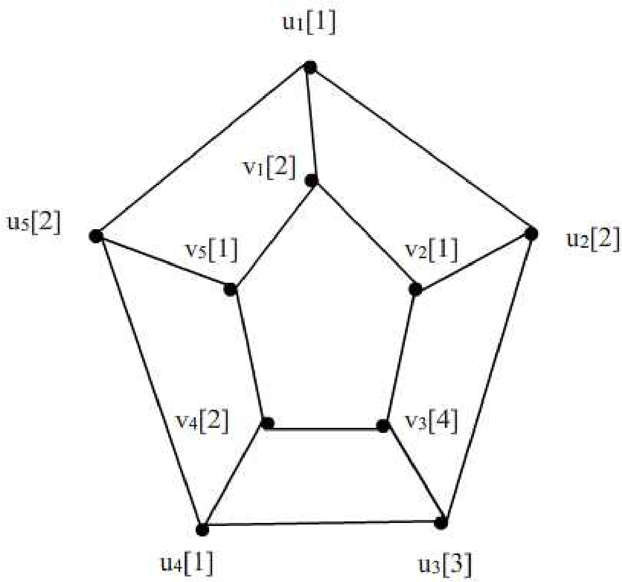

Figure 1. A Grundy 4-coloring of CL_5.

Case (i): : For n is odd.

A periodic 4-coloring may be defined by coloring indices modulo 3 as follows:

$$\begin{aligned}
&c_4 : v_i \quad \text{if } i \equiv 0 \pmod 3, \\
&c_3 : u_i \quad \text{if } i \equiv 0 \pmod 3, \\
&c_2 : u_i \quad \text{if } i \equiv 2 \pmod 3, \qquad v_i \quad \text{if } i \equiv 1 \pmod 3, \\
&c_1 : u_i \quad \text{if } i \equiv 1 \pmod 3, \qquad v_i \quad \text{if } i \equiv 2 \pmod 3.
\end{aligned}$$

This produces the Grundy sequence

$$c_1, c_2, c_3, c_4,$$

so $\Gamma(CL_n) \geq 4$ for odd n.

A sample coloring for CL_5 is shown below.

Case (ii): : For n is even.

A similar periodic coloring works for all even $n \geq 4$. One valid assignment is:

$$\begin{aligned} &c_4 : u_1,\ u_3, \\ &c_3 : v_1,\ v_3, \\ &c_2 : u_i\ (i \geq 4,\ i \text{ even}), \qquad v_i\ (i = 2 \text{ or } i \geq 5,\ i \text{ odd}), \\ &c_1 : u_i\ (i \geq 2,\ i \text{ odd}), \qquad v_i\ (i \geq 4,\ i \text{ even}). \end{aligned}$$

This again yields the Grundy sequence c_1, c_2, c_3, c_4 and therefore

$$\Gamma(CL_n) \geq 4 \qquad (n \text{ even}).$$

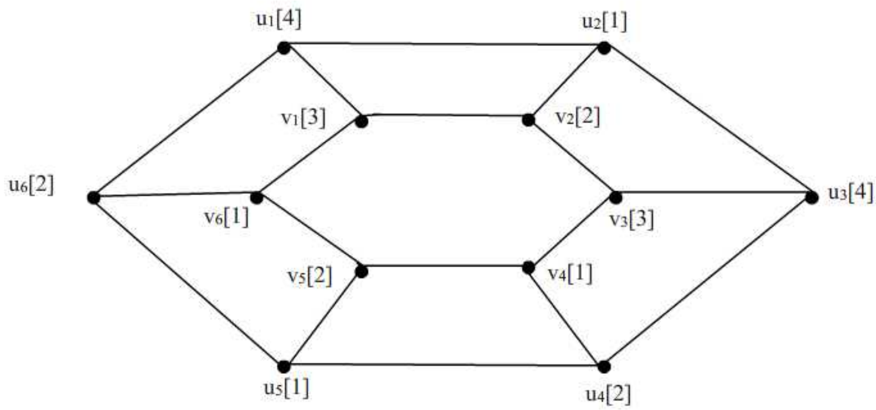

Figure 2. A Grundy 4-coloring of CL_6.

Assume, for contradiction, that $\Gamma(CL_n) \geq 5$. Then some vertex must receive color c_5, which requires neighbors realizing colors c_1, c_2, c_3, c_4 simultaneously. But each vertex in CL_n has degree

$$\Delta(CL_n) = 3,$$

so it can have at most three colors in its open neighborhood. Thus no vertex can receive color c_5, and hence

$$\Gamma(CL_n) \leq 4.$$

Combining both directions, we obtain

$$\Gamma(CL_n) = 4 \qquad \forall\, n \geq 3.$$

□

Theorem 3.2
For every integer $n \geq 4$, the Grundy chromatic number of the n–crossed prism graph R_n satisfies

$$\Gamma(R_n) = 4.$$

Proof
The crossed prism graph R_n has two n–cycles

$$U = \{u_1, u_2, \ldots, u_n\}, \qquad V = \{v_1, v_2, \ldots, v_n\},$$

and three edge sets:

$$E_1 = \{u_i u_{i+1},\, v_i v_{i+1} : 1 \leq i \leq n-1\} \cup \{u_1 u_n,\, v_1 v_n\},$$

$$E_2 = \{u_i v_{i+1} : i \text{ odd}\},$$
$$E_3 = \{u_i v_{i-1} : i \text{ even}\},$$

where indices are taken modulo n. Thus

$$E(R_n) = E_1 \cup E_2 \cup E_3.$$

Each vertex has degree

$$\deg(u_i) = \deg(v_i) = 3,$$

hence the general bound for Grundy colorings gives

$$\Gamma(R_n) \leq \Delta(R_n) + 1 = 4.$$

It remains to show that a Grundy 4-coloring exists for every $n \geq 4$.

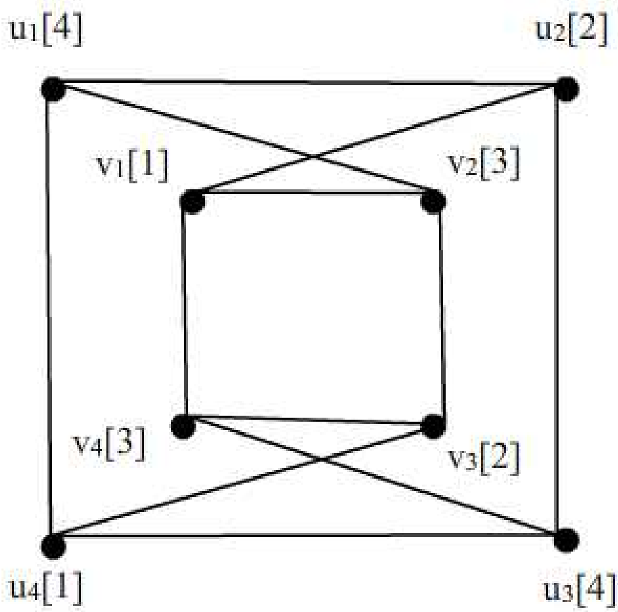

Figure 3. A Grundy 4-coloring of R_4.

Case (i): : $n \equiv 0 \pmod 4$.

A valid 4–coloring may be assigned periodically as follows:

$$\begin{aligned} &c_4 : u_i \quad (i \text{ odd}),\\ &c_3 : v_i \quad (i \text{ even}),\\ &c_2 : v_{4i-1},\ u_{4i-2},\\ &c_1 : v_{4i-3},\ u_{4i}, \end{aligned} \qquad 1 \leq i \leq n/4.$$

This produces the complete Grundy sequence

$$c_1, c_2, c_3, c_4,$$

so $\Gamma(R_n) \geq 4$ in this case.

Case (ii): : $n \equiv 2 \pmod 4$

A similar periodic construction yields a Grundy 4-coloring. One valid assignment is:

$$\begin{aligned}
&c_4 : u_1, \\
&c_3 : v_2,\ v_n,\ u_3, \\
&c_2 : u_i\ (i \geq 2 \text{ odd}), \quad v_i\ (i \text{ odd}), \\
&c_1 : u_i\ (i \geq 4 \text{ even}), \quad v_i\ (i \geq 4 \text{ even}).
\end{aligned}$$

Again this yields the Grundy sequence

$$c_1, c_2, c_3, c_4.$$

Assume for contradiction that $\Gamma(R_n) \geq 5$. Then some vertex would receive color c_5. By definition of Grundy

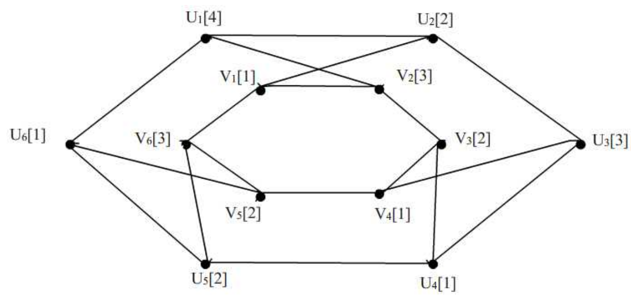

Figure 4. A Grundy 4-coloring of R_8.

coloring, that vertex must have neighbors colored with each of c_1, c_2, c_3, c_4. This requires degree at least 4. However, each vertex of R_n has degree

$$\deg(x) = 3,$$

so no vertex can realize four distinct smaller colors in its open neighborhood. This contradicts the assumption that a color c_5 is possible.

Thus,

$$\Gamma(R_n) \leq 4.$$

Combining the constructive lower bound and the degree upper bound gives

$$\Gamma(R_n) = 4 \qquad \text{for all } n \geq 4.$$

□

Theorem 3.3

The Grundy chromatic number of the line graph of the n-crossed prism graph is

$$\Gamma(L(R_n)) = 3.$$

Proof
Let $V(R_n) = \{u_i : 1 \le i \le n\} \cup \{v_i : 1 \le i \le n\}$, where the cycle edges are

$$E_1 = \{v_i v_{i+1} : 1 \le i \le n-1\} \cup \{v_1 v_n\},$$

the second cycle edges are

$$E_2 = \{u_i u_{i+1} : 1 \le i \le n-1\} \cup \{u_1 u_n\},$$

and the crossed edges are

$$E_3 = \begin{cases} u_i v_{i+1}, & i \text{ odd}, \\ u_i v_{i-1}, & i \text{ even}. \end{cases}$$

Thus,

$$E(R_n) = E_1 \cup E_2 \cup E_3.$$

The line graph $L(R_n)$ has vertex set

$$V(L(R_n)) = E(R_n),$$

and two vertices in $L(R_n)$ are adjacent when the corresponding edges of R_n share an endpoint.

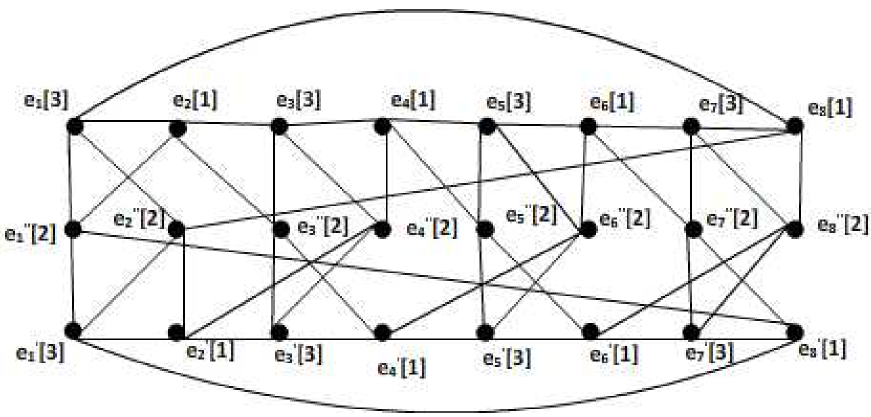

Figure 5. A Grundy 3-coloring of $L(R_8)$

We now exhibit a Grundy 3-coloring.

- Assign color c_3 to all vertices of $L(R_n)$ corresponding to edges $e_i \in E_1$ with i odd and $e'_i \in E_2$ with i odd.
- Assign color c_2 to all vertices corresponding to the crossed edges E_3.
- Assign color c_1 to the remaining cycle edges: $e_i \in E_1$ with i even and $e'_i \in E_2$ with i even.

This coloring is proper since consecutive edges in the underlying cycles alternate between even and odd indices, and the crossed edges meet both cycle edges of opposite parity. Moreover, each color class appears only when all smaller colors appear in its neighborhood, confirming that this is a Grundy coloring. Thus,

$$\Gamma(L(R_n)) \ge 3.$$

Now suppose $\Gamma(L(R_n)) > 3$. If some vertex receives color c_4, then by the definition of a Grundy coloring, its neighbors must contain all colors c_1, c_2, c_3. However, every edge in R_n is incident to at most two other edges in the same cycle and at most one crossed edge. Thus every vertex in $L(R_n)$ has degree at most 3. Hence no vertex can see three distinct colors in its neighborhood, and so no vertex can receive color c_4. Therefore,

$$\Gamma(L(R_n)) \leq 3.$$

Finally, assume $\Gamma(L(R_n)) < 3$. Then only colors c_1 and c_2 may be used. But in $L(R_n)$ there exist vertices that are mutually adjacent in paths of length three (coming from three incident edges of R_n), and such a structure demands at least three Grundy colors. Thus a 2-Grundy-coloring is impossible.

Combining the inequalities,

$$\Gamma(L(R_n)) = 3.$$

□

Theorem 3.4
For the antiprism graph Q_n on $2n$ vertices, the Grundy chromatic number is

$$\Gamma(Q_n) = \begin{cases} 3, & n = 3, \\ 5, & n > 3. \end{cases}$$

Proof
The antiprism graph Q_n has vertex set

$$V(Q_n) = \{v_i : 1 \leq i \leq n\} \cup \{u_i : 1 \leq i \leq n\},$$

with edges:

$$\begin{aligned} E(Q_n) = &\{v_i v_{i+1} : 1 \leq i \leq n-1\} \cup \{v_1 v_n\} \\ &\cup \{u_i u_{i+1} : 1 \leq i \leq n-1\} \cup \{u_1 u_n\} \\ &\cup \{v_i u_i : 1 \leq i \leq n\} \\ &\cup \{v_i u_{i+1} : 1 \leq i \leq n-1\} \cup \{v_n u_1\}. \end{aligned}$$

This construction produces two n-cycles linked by alternating diagonal edges. Since each vertex is adjacent to exactly four others, the degree satisfies

$$\Delta(Q_n) = 4,$$

and therefore the Grundy bound

$$\Gamma(Q_n) \leq \Delta(Q_n) + 1 = 5$$

holds for all n.

We now determine the exact value.

Case (i): : For $n = 3$ Assign the colors:

$$c_3 : u_1, v_3; \qquad c_2 : v_1, u_2; \qquad c_1 : v_2, u_3.$$

This is a valid Grundy coloring using 3 colors. Assume for contradiction that four colors are possible. Let a vertex receive color c_4. Then its neighbors must contain all colors c_1, c_2, c_3, but each vertex in Q_3 has degree 3, contradicting proper Grundy conditions. Thus,

$$\Gamma(Q_3) = 3.$$

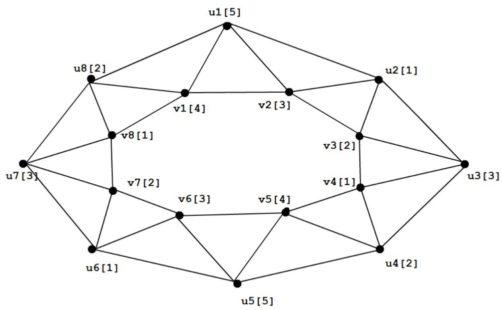

Figure 6. $\Gamma(Q_8)$

Case (ii): For $n \geq 4$.

For $n \geq 4$, we establish that the antiprism graph Q_n admits a Grundy coloring using five colors. To construct such a coloring, we color the vertices of the two n-cycles in a repeating pattern. More precisely, for $1 \leq i \leq \left\lfloor \frac{n}{4} \right\rfloor$, we assign

$$\begin{aligned} &c_5 : u_{4i-3}, \\ &c_4 : v_{4i-3}, \\ &c_3 : u_{4i-1},\ v_{4i-2}, \\ &c_2 : u_{4i},\ v_{4i-1}, \\ &c_1 : u_{4i-2},\ v_{4i}. \end{aligned}$$

This periodic assignment ensures that each color appears only after all smaller colors have already appeared

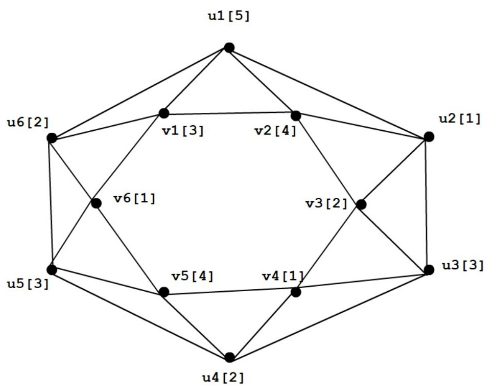

Figure 7. $\Gamma(Q_6)$

on the neighbors of the corresponding vertex, thereby forming a valid Grundy sequence

$$c_1,\ c_2,\ c_3,\ c_4,\ c_5.$$

Consequently,

$$\Gamma(Q_n) \geq 5.$$

To show that no sixth Grundy color can occur, suppose a vertex of Q_n is assigned color c_6. By definition of a Grundy coloring, this vertex must have neighbors colored with each of c_1, c_2, c_3, c_4, and c_5. However, every vertex in Q_n has degree 4, meaning that it can have at most four distinct colors in its neighborhood. This contradicts the requirement for a vertex to receive color c_6.

Hence, no Grundy coloring of Q_n uses more than five colors, and therefore

$$\Gamma(Q_n) = 5 \qquad \text{for all } n > 3.$$

□

4. Conclusion

In this work, we investigated the Grundy chromatic number for several important graph families, including the prism graph, the $n-$crossed prism graph, the line graph of the crossed prism graph, and the antiprism graph. Using constructive Grundy colorings and structural properties of each graph, we established exact values of their Grundy chromatic numbers and provided illustrative examples to demonstrate the achievable color sequences. These results contribute to a deeper understanding of Grundy colorings in structured graph classes and offer a foundation for further studies on dynamic colorings and chromatic bounds in complex network topologies.

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