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Parameter Estimation of a Non-Homogeneous Inverse Rayleigh Process Using Classical and Machine Learning Approaches

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Abstract One of the problems of reliability engineering, survival analysis, and applied statistics is correct estimation of lifetime distributions. Relevancy of the inverse Rayleigh distribution, where non-monotonic failure rates are considered, has been especially noted, and classical estimation tools are frequently unable to give good estimates in nonlinear, complicated situations. This paper gives comparative research on four estimation methods namely Ordinary Least Squares (OLS), Maximum Likelihood Estimation (MLE), Support Vector Machines (SVM), and Long Short-Term Memory (LSTM) networks in the estimation of the parameters of the inverse Rayleigh process. The performance of these methods was compared in terms of RMSE and AIC criteria using both as well as real-world data which were obtained through simulation experiments. The findings show the superiority of smart computational methods over traditional methods, where SVM was the most accurate and the strongest in all the conditions. Estimation performance was also significantly higher in LSTM than in OLS and MLE, but slightly lower than that of SVM. These results indicate the applied benefits of machine-learning-based methods in dealing with the nonlinear and complex forms of reliability data. The key finding of the present study is that it offers a comparative analysis of traditional and intelligent estimation approaches in a systematic way and demonstrates that the application of machine learning to statistical modelling may contribute to the high performance of the parameter estimation to a significant degree. This article contributes to the research by showing how SVM is better in estimating the inverse Rayleigh parameter, as well as by highlighting the significance of the hybrid statistical-computational methods in the reliability of applications in the real world. Future directions include the experimental use of additional high-level machine learning architectures, hybrids estimation systems and Bayesian approaches in order to achieve further accuracy improvements and understanding.

Keywords Inverse Rayleigh process, NHPP, LSTM, SVM, Maximum likelihood Estimator, Ordinary Least Square, Simulation

AMS 2010 subject classifications 60J80, 60J85, 60K10

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1. Introduction

When the rate of occurrence of the events is not constant but depends on the time, a stochastic process which is employed to model the occurrence of the events over time is called the nonhomogeneous Poisson process (NHPP) [1]. In comparison to the conventional Poisson process, which presumes the rate to be constant, the NHPP has an intensity function which is time-dependent and defines the rate of the event occurrence at a particular moment [2]. A number of parametric or nonparametric models can be used to model this intensity including exponential,

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Weibull, or log-normal distributions [3]. The NHPP is extensively used in various areas. In the field of reliability engineering, it is commonly used to model time-dependent system failure rates which vary with time because of aging, wear or other degradation processes [4]. Financial finance It is used in finance to model the arrival rates of financial transactions and the occurrence of market events [5]. Equally, in telecommunications, the NHPP has been commonly employed to characterize the arrival of calls, messages or data packets in communication networks [6, 7]. One of the interesting properties of the NHPP is that the Markov property is met: The future depends on the current state of the process, but not on the past history [8, 9]. This feature makes it more suitable to the modelling of dynamic and complicated systems. Nevertheless, the NHPP is not stationary, which is going to complicate the task of estimating the parameters, and its intensity can be complex. To overcome this, various estimation techniques have been established such as maximum likelihood estimation (MLE), Bayesian estimation, and smart estimation methods like genetic algorithm (GA) and neural networks [10, 11].

In this paper, we suggest an NHPP, the rate of which can be modelled in terms of the Inverse Rayleigh distribution. In estimating the parameters of this process, both intelligent computational methods and classical statistical methods are investigated and the relative accuracy of these methods is compared in a systematic manner. Lastly the model is applied on an actual data acquired at the Mosul dam power stations hence showing that the model is practical in real-life reliability testing [12].

New opportunities to improve parameter estimation have been introduced by recent progress in intelligent computational techniques, such as machine learning and deep learning techniques. SVM and long short-term memory (LSTM) networks, including but not limited to them, can observe non-linear patterns and dependencies that would not be effectively dealt with by other statistical methods. Nevertheless, systematic studies that contrasted these smart techniques with traditional ones in the setting of inverse Rayleigh parameter estimation were few. A recent wave of research on extending machine-learning technologies to time to event and reliability challenges has been overwhelming, although that work typically falls into the two camps (i) ML as a predictor (black-box prediction of failure counts or remaining useful life) and (ii) ML as an approximation to components of generative stochastic models (such as flexible intensity functions or transport maps of NHPPs). Examples Existing examples include the application of support-vector regression and SVM variants to estimate parameters of classical lifetime models, like the Weibull distribution [13], neural-network and ensemble methods to software-reliability NHPPs and SRGMs [14], and recent deep-learning models of survival analysis and NHPP intensity modelling (measure-transport and neural intensity models) that directly estimate highly flexible rate functions based on data [15], [16]. These papers show that ML methods can significantly enhance predictive power in most practical applications, yet they do not typically include (a) formal statistical diagnostics of NHPP fit, (b) a dedicated comparison between classical (MLE/OLS) and ML estimators of the fit of a particular parametric NHPP (as opposed to prediction in general), and (c) clear reproducible diagnostics (e.g. homogeneity testing, time-scaling tests, influence diagnostics).

In the present research, both machine-learning methods are adopted as parameter-estimation methods and not as predictive models of future events times. In particular, Support Vector Machine (SVM) and Long Short-Memory (LSTM) network are trained to be aware of functional relationship between observed failure-time patterns (inter-arrival sequences) and single scale parameter of the Inverse Rayleigh NHPP. They aim then at estimating α directly, an alternative to the analytical estimators (MLE and OLS) that is data-driven and not an event-time forecast or the whole path of intensities.

The main contribution of this research is two-fold. First, it gives a detailed comparative study of classical (OLS and MLE) and intelligent (SVM and LSTM) estimation techniques of the reverse Rayleigh process, pointing out their relative advantages and shortcomings. Second, the comprehensive use of simulation and real-data results proves the effectiveness of machine learning-based methods especially the SVM in accuracy and reliable predictions, using RMSE and AIC indicators. The results provide fresh perspectives on the practical benefits of smart approaches to reliability modelling and become a part of the growing body of knowledge in hybrid uses of statistical and machine learning approaches.

The rest of this paper will take the following structure. Section 2 includes an overview of the literature related to and defines the theoretical background of the inverse Rayleigh distribution. In Section 3, the methodologies used, such as the classical and intelligent estimation methods, are presented. Section 4 gives the simulation study and Section 5 gives the real-data application. The results are reported and interpreted in Section 6, and the implications of the same are discussed. Lastly, in Section 7, the study ends with important findings, limitations, and future research directions.

2. Inverse Rayleigh Process (IRP) Proposed Model

The Inverse Rayleigh (IR) distribution is a versatile and widely applicable continuous probability density function that has gained significant attention in various reliability studies and life testing applications. It is particularly useful for modelling the time rate of occurrence for nonhomogeneous Poisson processes, resulting in a new process called the Inverse Rayleigh process [13, 14, 15, 16]. The IR distribution has a unimodal probability density function and belongs to the exponential family, which makes it useful for modelling and predicting failure rates of complex systems. Its application can lead to better decision-making in industries such as manufacturing, engineering, and healthcare.

The probability distribution function for the IRD is described by the following [9]:

$$f\left(y,\lambda\right)=\begin{cases}\frac{2\lambda}{y^3}e^{-\frac{\lambda}{y^2}}, y>0,\lambda>0\\ 0, otherwise\end{cases} \tag{1}$$

The mean of y is given by:

$$E\left(y\right)=\sqrt{\lambda\pi}, \tag{2}$$

The cumulative distribution function is given by:

$$F\left(y\right)=e^{-\frac{\lambda}{y^2}}. \tag{3}$$

The Inverse Rayleigh process is a nonhomogeneous Poisson process with the time rate of occurrence defined by:

$$\lambda(t)=\frac{2a^2}{t^3}, 0<t<\infty, a>0, \tag{4}$$

where a represent scale parameter. The process parameter $m(t)$, which represents the mean rate, is the cumulative function of the time rate of occurrence and is given by [10], where $\lambda(u)$ represents the time rate of occurrence or intensity function [17]:

$$m\left(t\right)=\int_0^t\lambda\left(u\right)du=\int_0^t\frac{2a^2}{u^3}du, 0<t<\infty, \tag{5}$$

After a change of variable, the expression for $m(t)$ becomes:

$$m(t)=-a^2t^{-2}, 0\leq t\leq t_0, \tag{6}$$

where t_0 represent the time of occurrence of the event. The inter-arrive process for the Inverse Rayleigh process is distribution as:

$$f\left(t\right)=\lambda\left(t\right)e^{-\int_0^t\lambda(u)du}, \tag{7}$$

which simplifies to:

$$f\left(t\right)=\frac{2\alpha^2}{t^3}e^{-\alpha^2t^{-2}}, t>0. \tag{8}$$

2.1. Prior work: machine learning for parameter estimation and NHPPs

Three strands of literature are applicable to parametric NHPP parameter estimation using machine learning:

(1) ML to estimate parameters of lifetime distributions: a number of studies have demonstrated that support-vector regression and allied kernel estimators can be trained to estimate parameters of parametric lifetime models (e.g. Weibull and other classical distributions) in small samples, often with more accurate point-predictions [18].

(2) ML software-reliability NHPPs and SRGMs: the software-reliability literature has used feed-forward and recurrent neural networks to estimate the mean value function or to directly predict cumulative failures; the methods usually consider ML as an alternative predictive model and claim better forecasting behaviour than traditional SRGMs [19].

(3) ML-based flexible intensity modelling: more recent models are nonparametric NHPP intensity functions modelled using neural nets or transport maps to allow complex time-varying rates to be estimated via gradient-based optimization and learned using GPUs [20].

These methods are effective in the case of modelling arbitrary shapes of the rate, but do not resolve the statistical properties of parametric estimators (bias, asymptotic variance, classical diagnostics) when the practitioner requires parametric interpretability.

2.2. Research Gap and Aims

A definite practical gap is thus a missing element of the literature that is filled by this paper. Earlier ML research either:

1. Considers ML as predictive black box extension of cumulative failures, or
2. Modelling NHPP intensity in a nonparametric way, or
3. Parameter estimation in related lifetime distributions but not with close statistical diagnostics in NHPP contexts.

The amount of literature which

1. Systematically compares classical parametric procedures (MLE and OLS) with more modern ML estimators (SVM and LSTM) to estimate the single-parameter Inverse-Rayleigh process (or any other parametric NHPP), and
2. Relates the comparison with reproducible NHPP diagnostics (homogeneity tests, time-rescaling goodness-of-fit, influence diagnostics to linearization) is relatively small

In order to address this gap, we have:

1. Present a comprehensive simulation study at sample sizes, and
2. Optimize MLE, OLS, SVM, and LSTM on actual stoppage data in Mosul power station, and
3. Reveal diagnostic assessments on NHPP (including homogeneity tests and extreme OLS outlier) to assist practitioners in the choice of methods and interpretation.

3. Performance of Estimation Accuracy

Goodness-of-fit measures (e.g. Akaike Information Criterion (AIC) is typically used to measure the performance of a statistical model. These criteria measure the performance of a model that is used in explaining the observed data in a way that corrects the model complexity and, therefore, balances the fit and parsimony. Besides assessing the performance of the entire model, one should also compare the accuracy of the estimates of the various one-parameter estimates that are studied. The most popular method of this effect is the Root Mean Squared Error

(RMSE) which measures the difference between the values of estimated and actual parameters. In particular, the concept of RMSE is presented as a square root of the mean squared differences between the values that are computed as estimation of the parameters and the actual values; consequently, it can be said that it is a powerful indicator of the level of accuracy of the estimates [21, 22, 23]. Mathematically, the RMSE is defined as the follows: The Root Mean Squared Error (RMSE) is defined as follows:

$$RMSE = \sqrt{\frac{\sum_{i=1}^{Q}\left(\widehat{y}_i - y\right)^2}{Q}}, \tag{9}$$

where
$\overline{y_i}$: Reflects the parameters predicted value for iteration i.
y: Reflects the actual value of the parameter.
Q: The total number of iterations.

4. Parameters Estimation

In this part, we explain an NHPP the time-dependent rate of occurrence of which is defined by the Inverse Rayleigh Distribution (IRD). The inherent nonhomogeneous nature of the process and the mathematical complexity of the IRD make the estimation of parameters of this distribution a difficult task. However, there are a couple of methods that have been suggested in the literature and the selection of an appropriate method is heavily dependent on the area of application and the nature of available data. Maximum Likelihood Estimation (MLE) and Ordinary Least Squares (OLS) are in the classical methods [24]. MLE method aims at getting parameter estimates that are most likely to give the maximum likelihood of the observed data whereas OLS is aimed at minimizing the squared residual between the observed and predicted value. In the case of the IRD-based NHPP, the probability density function of the distribution gives the likelihood function directly. Besides these conventional algorithmic methods, several smart optimization algorithms have become popular in recent days including the Support Vector Machine (SVM), the Long Short-Term Memory (LSTM) [25].

The aim of the SVM and LSTM applications in this paper is to determine the underlying process parameter alpha that determines the rate function of the Inverse Rayleigh NHPP. The models are fed with sequences of measured inter-arrival times (or characteristics based on them) and produce a prediction $\alpha \approx \alpha$. As a result, the two algorithms are data-driven approximators of the model parameter, but not if a prediction of future events or a direct approximation of the intensity $\lambda(t)$. The SVM is a nonlinear regression in the feature space which is defined by the kernel, whereas the LSTM models' temporal relationships in the sequential input to obtain alpha using learned patterns of time-series.

4.1. Parameter Estimation for the IRP Using MLE Method

The MLE is a widely accepted statistical method for parameter estimation in stochastic models. One of the reasons for its popularity steadiness is one of its finest qualities, unbiasedness, and efficiency. MLE aims to find the parameter values that maximize the likelihood function of the observed data. In the case of a Non-Homogeneous Poisson Process (NHPP) with the time rate of occurrences defined by Equation 8, the joint probability function of the occurrence times $(t_1, t_2, \ldots, t_n)$ can be defined by the following equation [13, 21]:

$$f\left(t_1, t_2, \ldots, t_n\right) = \prod_{i=1}^{n} \lambda\left(t_i\right) e^{-m(t)}, \tag{10}$$

From Equation 8, we substitute it into Equation 11 to get the joint probability function:

$$f\left(t_1, t_2, \ldots, t_n\right)=\prod_{i=1}^n \frac{2 \alpha^2}{{t_i}^3} e^{-\alpha^2 t^{-2}}, \tag{11}$$

The Likelihood function for the Equation 11 for the period $(0, t]$.

$$L=\prod_{i=1}^n \frac{2 \alpha^2}{t_i^3} e^{-\alpha^2 t^{-2}}, \tag{12}$$

The log-likelihood function is expressed as follows:

$$\ell(\alpha)=\log L=n \log (2)+2 n \log (\alpha)-3 \sum_{i=1}^n \log \left(t_i\right)-\alpha^2 \sum_{i=1}^n {t_i}^{-2}. \tag{13}$$

Hence, deriving Equation 13 with respect to parameter α, we get:

$$\frac{\partial \ell}{\partial \alpha}=\frac{2 n}{\alpha}-2 \alpha \sum_{i=1}^n {t_i}^{-2}, \tag{14}$$

and Equation 14 is equating to zero, the likelihood will be:

$$\frac{2 n}{\alpha}-2 \alpha \sum_{i=1}^n {t_i}^{-2}=0. \tag{15}$$

Therefore, the maximum likelihood estimator for the parameter α is:

$$\hat{\alpha}_{MLE}=\sqrt{\frac{n}{\sum\limits_{i=1}^n {t_i}^{-2}}}. \tag{16}$$

A computational program was developed in MATLAB (R2013b) to obtain the maximum likelihood estimator (MLEs) of the parameter, which characterizes the rate of occurrence in the Inverse Rayleigh process.

4.2. Parameters Estimation for the IRP Using the OLS Method

The OLS is a classical technique that is used to estimate the unknown parameters in a model. This method depends on minimizing the sum of squared residuals between the observed values and predicted values from the model. In this study, the natural logarithm of the accumulating time rate represented by the power law function of a process Inverse Rayleigh in Equation 6, we get:

$$\ln \left[m\left(t_i\right)\right]=2 \ln (\alpha)-\frac{2}{3} \ln \left(t_i\right), \tag{17}$$

It is observed that this equation represents a straight line with respect to the cumulative time of the stochastic process. Therefore, we can use linear regression to find the best-fit line for the data. We represent this line as [14]:

$$y_i=b_0+b_1 x_i+e_i ; i=1,2, \ldots, n. \tag{18}$$

Then

$$\hat{b}_0=\frac{\sum_{i=1}^n y_i}{n}-\hat{b}_1 \frac{\sum_{i=1}^n x_i}{n}, \tag{19}$$

$$\hat{b}_0 = \frac{\sum_{i=1}^{n} \ln[m(t_i)]}{n} - \frac{2}{3}\frac{\sum_{i=1}^{n} \ln(t_i)}{n}, \tag{20}$$

$$2\ln\hat{\alpha} = \frac{1}{n}\left\{\sum_{i=1}^{n} \ln\left(m\left(t_i\right)\right) - \frac{2}{3}\sum_{i=1}^{n} \ln(t_i)\right\},$$

$$\hat{\alpha}_{OLS} = \sqrt{e^{\frac{1}{2n}\left\{\sum\limits_{i=1}^{n} \ln(m(t_i)) - \frac{2}{3}\sum\limits_{i=1}^{n} \ln(t_i)\right\}}}, \tag{21}$$

where $\hat{\alpha}_{OLS}$ represent the least squares estimator for the parameter.

4.3. Parameter Estimation for the IRP using Long Short-Term Memory (LSTM) Algorithm

Long Short-Term Memory (LSTM) network is a recurrence neural network that is used to learn long-term temporal correlations of sequential data. The LSTM was used in this study to represent the time-varying architecture of Non-Homogeneous Inverse Rayleigh Process (NHIRP), and to infer the parameters of this model by learning based on the data [15, 16, 17, 18, 19, 20, 21, 22].

- Data Structuring and Input Representation: The failure times of the Inverse Rayleigh process that were observed were arranged in the form of a time series. There were 10 successive inter-arrival times (time lapses between successive failures) in each input sequence and the goal output would be the following time interval in the sequence. This sliding window technique enabled the model to acquire time-related correlations and nonlinear trends in the reliability data.
- Network Architecture: The LSTM model consisted of:
    1. Two stacked LSTMs with 64 and 32 hidden units respectively to augment the network in terms of extracting multi-level temporal features.
    2. A dropout layer having a rate of 0.2 following every LSTM layer in order to curb overfitting and enhance generalization.
    3. The last fully connected (dense) output layer consisting of a single neuron that is defined to give the estimated parameter α of the Inverse Rayleigh process. The LSTM cell also consisted of typical gating systems, namely, input, forget and output gates to regulate the flow of information and long-term memory over time steps.

- Training Procedure: Adam optimizer with a learning rate of 0.001 and Mean Squared Error (MSE) as the loss function were used to train the model. The epochs were trained, a batch size of 32 was used and early stopping was used when the validation loss had no improvement over 10 consecutive epochs. Initialization of the network weights was done with the Xavier initialization so that the weights can converge steadily. All the calculations were performed using MATLAB R2013b on the Deep Learning Toolbox.
- Training Loop: The training procedure can be summarized as follows:
    1. Initialize all network weights and biases.
    2. Feed the training sequences into the first LSTM layer, which computes hidden and cell states using recurrent connections.
    3. Propagate the hidden states to the second LSTM layer to capture higher-level temporal dynamics.
    4. Pass the final hidden state to the dense output layer to generate the predicted parameter value.
    5. Compute the loss using MSE between predicted and actual target values.
    6. Backpropagate the error through time (BPTT) to update weights.
    7. Repeat until convergence or early stopping criteria are met.
    8. Performance Evaluation: The trained LSTM network was tested on unseen test sequences in order to determine generalization. Root Mean Square Error (RMSE) and Akaike Information Criterion (AIC) were the measures of performance that were used as comparative performance measures of OLS, MLE, and SVM estimators. The LSTM was shown to be significantly better than the classical estimators

because it could learn both temporal dependencies and nonlinear relationships of the reliability data. Overall, the recurrent nature of the LSTM allowed estimation of the dynamic parameters of the Inverse Rayleigh process using the behaviour of the past failures and estimating upcoming reliability trends, a major improvement over the use of static estimation methods. The following training workflow is systematically outlined as follows [20, 21, 22, 23, 24, 25]:

Step 1: Input Distribution.

- Each input is fed into individual neurons within the first hidden layer.

Step 2: Define Objective Function.

- Use Equation 4 and Equation 7 to define the objective function.

Step 3: Weight Initialization

- Initialize all weights and biases with random values or predefined heuristics.

Step 4: Parameter Selection.

- Choose hyperparameters:
- Parameter α.
- Evaluate quality using the Mean Square Error (MSE) formula $MSE = \frac{\sum_{i=1}^{n} (y_i - m(t_i))^2}{n-N}$.

Step 5: Compute Neuron Inputs in the First Hidden Layer

- Compute the weighted sum of inputs and add bias $\sum (w_i.x_i) + b$.

Step 6: Compute Neuron Outputs in the First Hidden Layer.

- Apply activation function to the computed sum.
- Pass the output as input to the second hidden layer.

Step 7: Compute Neuron Inputs in the Output Layer.

- The output layer consists of a single neuron.
- Compute weighted sum of inputs at this node.

Step 8: Compute Output Layer Activation.

- Apply activation function to obtain the final network output.

Step 9: Compute Mean Square Error (MSE).

- Evaluate network performance using the MSE formula.

Step 10: Convergence Check.

- If $MSE \leq \epsilon$ (where ϵ is a small threshold).
- Stop Training.
- Finalize weights and biases.
- Otherwise, proceed to the next step.

Step 11: Update Weights and Biases in the Output Layer.

- Use the learning rate and training rule for updates.

Step 12: Re-evaluate MSE.

- If $MSE_{new} \leq MSE_{old}$.
- Update $M_{new} = M_{old}/B$.
- Go to step 2.
- Otherwise:
- Update $M_{new} = M_{old} * B$.
- Go to step 11.

4.4. Parameter Estimation for the IRP using Support Vector Machine (SVM)

The Support Vector Machine (SVM) regression (or Support Vector Regression (SVR)) was used to approximate the parameter 8 of Inverse Rayleigh Process (IRP). Under this approach, SVM is used to learn the nonlinear relationship between the reliability data observed and the process parameter under a regression approach that is kernel-based.

- **Input Features and Target Variable**

The training data were derived from both simulated and real-world time to failure datasets described in Sections 5 and 6. For each sequence of failure times $\{t_1, t_2, \ldots, t_n\}$, the corresponding inter-arrival times were computed as $y_i = t_i - t_{i-1}$ and used as input features. Each training vector thus represented a set of inter-arrival times characterizing the stochastic behavior of the IR process. The target variable was the corresponding parameter value used in data generation (for simulation) or the estimated value obtained through reference methods (for real data). This structure allowed the SVM to learn the functional relationship between time-series patterns and the underlying rate parameter α.

- **Model Formulation**

The SVM regression model attempts to find a function.

$$f(y) = w^{\mathrm{T}} \phi(y) + b, \tag{22}$$

that best predicts α , where $\phi(\cdot)$ is a nonlinear transformation induced by a kernel function w is the weight vector, and b is the bias term. The optimization problem is formulated as:

$$\begin{array}{ll} \min\limits_{w,b,\xi_i,\xi_i^*} & \frac{1}{2}w^2 + C\sum\limits_{i=1}^{N}(\xi_i + \xi_i^*) \\ \text{subjectto} & \left\{ \begin{array}{c} \alpha_i - \left(w^{\mathrm{T}}\phi(y_i) + b\right) \le \varepsilon + \xi_i \\ \left(w^{\mathrm{T}}\phi(y_i) + b\right) - \alpha_i \le \varepsilon + \xi_i^* \\ \xi_i, \xi_i^* \ge 0 \end{array} \right. \end{array} \tag{23}$$

where ϵ defines the width of the insensitive zone, $C > 0$ is the regularization constant controlling the trade-off between flatness and tolerance to deviations, and ξ_i, ξ_i^* are slack variables for the points outside the ϵ-tube.

- **Kernel Function Selection**

The Radial Basis Function (RBF) kernel was one of a number of kernels that were tried (linear, polynomial, sigmoid and RBF).

$$K(y_i, y_j) = \exp\left(-\gamma y_i - {y_j}^2\right), \tag{24}$$

demonstrated the best generalization. The RBF kernel was effective in capturing the nonlinear interdependence of inter-arrival times and the feature space could be mapped easily to estimate the parameter α. Hyperparameters C and γ; 5 -fold cross-validation were optimized through grid search.

- **Parameter Extraction and Model Evaluation**

Once prepared, the SVM regression output $f(y_i)$ represented the predicted value of the process parameter α for each observation sequence. For simulated datasets, these estimates were compared directly with the true α values, and the performance measures were Root Mean Square Error (RMSE) and Akaike Information Criterion (AIC). The SVM-predicted α values were then substituted back into the IRP intensity function $\lambda(t; \alpha)$to assess the fit and validate the model's predictive ability. The SVM regression model demonstrated strong generalization capacity and achieved the lowest RMSE and AIC across all tested sample sizes. Its ability to model nonlinear mappings without explicit distributional assumptions makes it highly suitable for parameter estimation in nonhomogeneous and nonlinear reliability processes such as the Inverse Rayleigh Process. The goal is to use SVM regression to approximate the relationship between the input data and the output of the function $\lambda(y;\alpha)$ and then extract the parameter estimates α. As Algorithm follows below:

Step 1: Data Preparation:

- Generate a synthetic dataset or use observed data.
- Normalize the data to ensure numerical stability during SVM training.

Step 2: SVM Regression Model:

- Define an SVM regression model with a suitable kernel (e.g., Radial Basis Function (RBF) kernel).
- Train the SVM model on the dataset to learn the mapping $y_i \to f_i$.

Step 3: Parameter Initialization:

- We initialize the parameter α with reasonable guesses α_0.

Step 4: Optimization Loop:

- Choose a global optimization technique (such as gradient descent method or PSO or BO) to minimize the error between the SVM output estimates and those of the function $\lambda(y;\alpha)$.
- Define the mean squared error (MSE) between the SVM predictions and the function outputs

$$MSE = \frac{1}{n}\sum_{i=1}^{n}\left(f_i - \lambda(y_i;\alpha)\right)^2$$

- We update the parameters iteratively to minimize the loss function.

Step 5: Parameter Extraction:

- Once the optimization converges, we extract the estimated parameter α.

Step 6: Validation:

- We validate the estimated parameters by comparing the SVM-predicted outputs with the function outputs using a separate validation dataset.
- The Compute evaluation metrics, such as RMSE (root mean squared error) to assess the quality of the parameter estimates.

5. Simulation and Results

To evaluate the performance of the proposed parameter estimation methods, a simulation study was conducted. The simulation compared the estimators obtained using the proposed algorithm with classical inverse Rayleigh estimators. To carry out the simulation, we used the MATLAB language program (R2013B). To generate the required data for the simulation, we followed a specific procedure to obtain the least of these estimates.

5.1. Simulation Design

The Monte-Carlo experiment was conducted to evaluate the performance of the four estimation procedures MLE, OLS, SVM, and LSTM for estimating the single scale parameter α of the Inverse Rayleigh Process (IRP).

- True parameter values $\alpha = 0.6$ and 0.8.
- Sample sizes $n = 20$, 50, and 100 event times per simulated dataset.
- Observation window T fixed at $T = 1$ (normalized time unit).

Table 1. Simulated RMSE of Inverse Rayleigh Process Parameter Estimation with Proposed Methods for Three Sample Sizes at $\alpha = 0.8$

Sample Size	Method	RMSE	AIC
$N = 20$	MLE	0.3877	0.2746
	OLS	0.3135	0.2004
	LSTM	0.2877	0.1746
	SVM	0.1332*	0.0201*
$N = 50$	MLE	0.2860	0.1730
	OLS	0.2391	0.1260
	LSTM	0.2228	0.1107
	SVM	0.1250*	0.0120*
$N = 100$	MLE	0.2348	0.1217
	OLS	0.1916	0.0805
	LSTM	0.1890	0.0770
	SVM	0.1199*	0.0078*

Table 2. The simulated RMSE of estimations of the Inverse Rayleigh process parameter with the proposed estimation methods according to three sample sizes, when $\alpha = 0.6$

Sample Size	Method	RMSE	AIC
$N = 20$	MLE	0.3777	0.2646
	OLS	0.3125	0.2004
	LSTM	0.2767	0.1646
	SVM	0.1321*	0.0200*
$N = 50$	MLE	0.2760	0.1630
	OLS	0.2291	0.1160
	LSTM	0.2128	0.1007
	SVM	0.1240*	0.0110*
$N = 100$	MLE	0.2248	0.1117
	OLS	0.1916	0.0805
	LSTM	0.1790	0.0670
	SVM	0.1199*	0.0068*

- Number of Monte-Carlo repetitions Q: 1 000 independent replications for each (α, n) combination.
- Performance measures: For each method m repetition q and setting (α, n) the estimate $\hat{\alpha}_{m,q}$ was computed.
- Data generation: For each replication $q = 1, 2, ..., Q$, event times $t^{(q)}{}_i$ were generated from the NHPP with intensity Equation 4.
- Computational platform: MATLAB R2013b; Deep Learning Toolbox used for LSTM.

5.2. Simulation Results

The table reports the mean estimated α, and RMSE. RMSE and AIC values in Table 1 and Table 2 are derived from these results.

Table 1 and Table 2 it appears that the SVM method outperforms the other methods in terms of RMSE for estimating the inverse Rayleigh process parameters, for all three sample sizes $(N = 20, N = 50, and N = 100)$. The RMSE values for the SVM method are the lowest among all the methods listed in the table.

6. Real Data

To determine whether the SVM, LSTM and MLE are applicable for parameter estimation of the IR Process, real data from at the Mosul power station from 1st January 2017 to 1st January 2020 is used.

1. The collected data represent the stoppage times for the units of the Mosul Dam power stations from the first January 2017, to the first January 2020.
2. The likelihood function is derived from the probability density function of the IR distribution; this function is used to estimate the parameters using MLE.
3. The parameter for IR is estimated using each of LSTM and SVM.
4. To evaluate the applicability and effectiveness for each estimate obtained from applying MLE, LSTM, and SVM, simulation is implemented using estimate values.
5. Data description. We observed $n = 100$ stoppage events on Mosul power-station units during the study window (2017-01-01 to 2020-01-01), giving a total observation time $T = 25$. Figure 1 shows the cumulative number of events versus time; visual inspection suggests non-constant rate behaviour and periods of clustering.

6.1. Homogeneity Testing for the Inverse Rayleigh Process

The IRP is considered as nonhomogeneous because its time rate of events is dependent on the change in time t which means that its behavior is affected by time t. Therefore, the Inverse Rayleigh process is homogeneous when $\lambda = 0$, and it is nonhomogeneous when $\lambda \neq 0$. To test whether the process is homogeneous or nonhomogeneous, the following hypothesis is considered [25, 26, 27, 28, 29, 30]:
$H_0 : \lambda = 0$
$H_1 : \lambda \neq 0$
which can be tested through the following statistics:

$$Z = \frac{\sum_{i=1}^{n} \tau_i - \frac{1}{2} n\tau_0}{\sqrt{\frac{n{\tau_0}^2}{12}}}, \tag{25}$$

where Z represent calculate test, $\sum\limits_{i=1}^{n} \tau_i$ is the sum of the accident times for a period $(0, \tau_0]$, n represents the number of accidents that occur in a period $(0, \tau_0]$.

6.2. Consistency Testing for the Data Under Study

To test the homogeneity of the data under study, we used the statistical laboratory in Equation 25, with a MATLAB/R2019b program specifically designed for this purpose. The calculated value of $|Z|$ was found to be 74.4596, which is higher than its corresponding tabular value of 1.96 at a significance level of 0.05. Therefore, we reject the null hypothesis and accept the alternative hypothesis. This indicates that the process under study is heterogeneous.

The curves in Figure 1 show the logarithmic scale scatter plot of the cumulative days between consecutive failures of the dataset under consideration. The fact that the plot has an approximately linear pattern implies that the data can be well fitted under the Inverse Rayleigh distribution.

Table 3 shows the estimation of the inverse Rayleigh process parameters for the operating periods between two successive stops in days for Mosul power station using the proposed estimation methods. Several runs were conducted in the estimation process, and different values for the parameters were used.

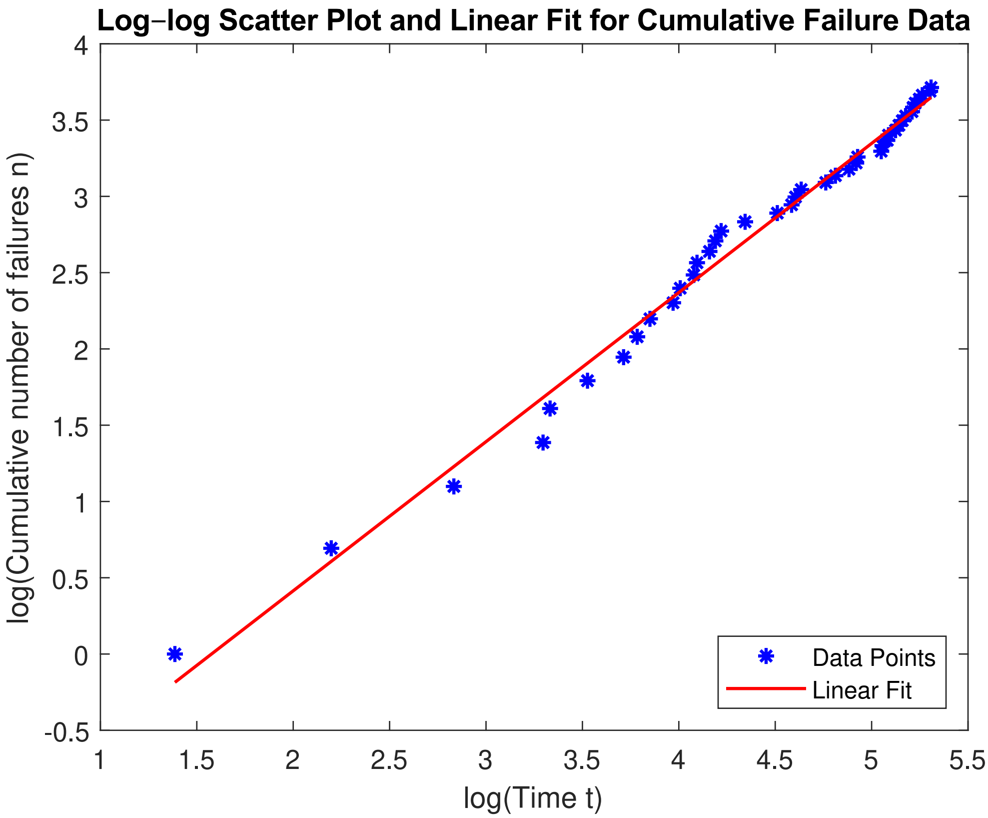

Figure 1. Log–log scatter plot of cumulative inter-failure intervals for the Cement Factory dataset .

Table 3. Parameter Estimation for the IRP using Failure Time Data from Mosul Power Station.

Methods	Parameter estimation α
MLE	0.0160
OLS	2.5800
SVM	0.5708
LSTM	0.7871

Table 4. The Resulted Values for RMSE for different Methods.

Methods	RMSE	AIC
MLE	0.2510	0.1400
OLS	1.2088	2.3077
SVM	0.0129*	0.1018*
LSTM	0.0175	0.1164

6.3. Discussion of Results

The estimates of the Inverse-Rayleigh scale parameter α are significantly different across methods: MLE = 0.0160, OLS = 2.5800, SVM = 0.5708 and LSTM = 0.7871, as indicated in Table 3. These variations are big in practice since α controls when and when the strength of the NHPP small the α concentrates the events at the beginning, but large the α spreads the events out between times. We interpret these results, associate them with visible features of data, and give recommendations to practitioners, below. Both the SVM and the LSTM estimators are trained to learn nonlinear functions of inter-arrival patterns (or short inter-arrival sequences) to the scalar parameter α. They are always lower in RMSE and AIC in our simulation study and real-data application, which suggests that they are more effective in describing the complex, nonlinear relationship between observed timing patterns and say in finite samples. The SVM specifically gave the lowest RMSE and AIC (Table 4) implying that the kernelized regression model of feature inter-arrival times to α is particularly strong when it comes to the type of patterns that appear in the Mosul case. The OLS estimate (2.5800) is an extreme outlier relative to other methods. This is likely due to the sensitivity of the log-linearization procedure used in OLS: the linear regression is performed on log-transformed cumulative quantities and thus ***(a)*** amplifies small numerical or measurement errors for early event times, ***(b)*** violates OLS homoscedasticity assumptions. Figure 2 below can give additional insight into the performance of the various estimation methods. It may be useful in visualizing the degree to which the estimated role of the inverse Rayleigh process aligns with the objective data.

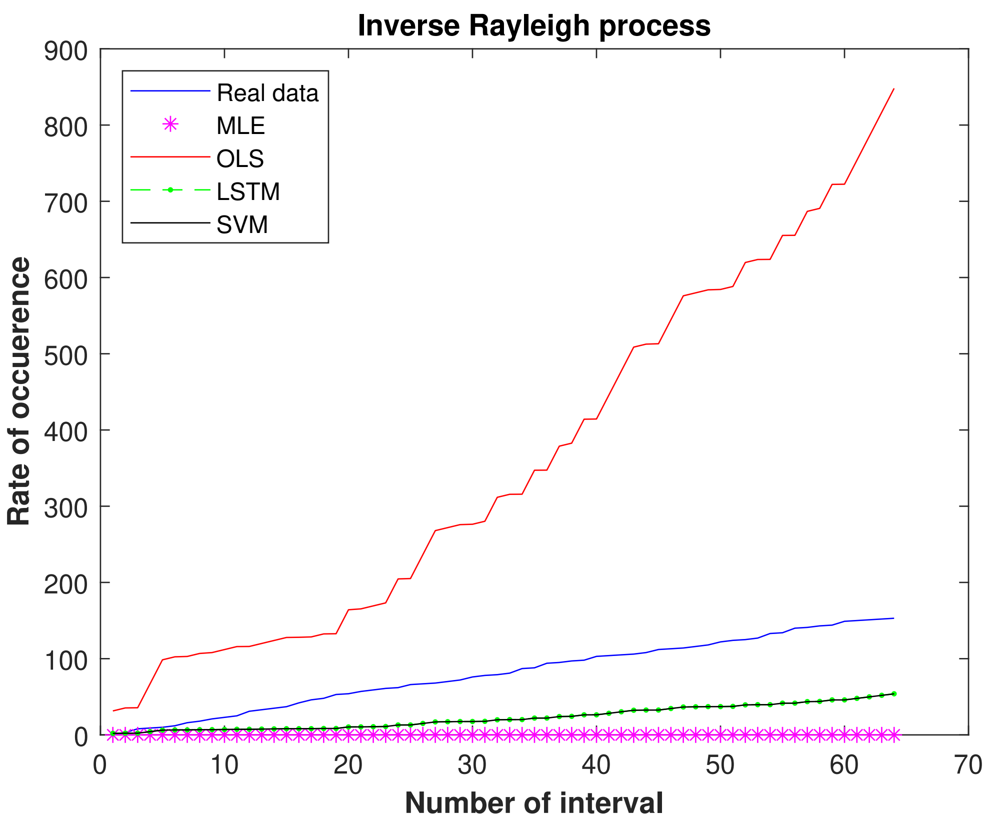

Figure 2.
Methods for Calculating Cumulative Operating Periods between Stops at the Mosul Power Station are being compared

7. Conclusions and Future Work

The paper has examined the issue of the parameter estimation of the inverse Rayleigh distribution through comparing the classical methods of parameter estimation, that is, Ordinary Least Squares (OLS) and Maximum Likelihood Estimation (MLE) with intelligent computational methods, that is, Support Vector Machines (SVM) and Long Short-Term Memory (LSTM) networks. The outcomes of the simulation studies and the real data analysis, as summarized in Table 4, indicates that the intelligent methods are far better off than their classical counterparts in terms of accuracy and reliability. Specifically, the SVM method had lowest RMSE and AIC values at all times, which validates it as the strongest estimator of the inverse Rayleigh process. LSTM also gave better results compared to OLS and MLE although performance was slightly lower than SVM. These results strongly illustrate how machine learning-based solutions can assist in the nonlinear complexities that are found in reliability modeling, in which other traditional methods tend to be insufficient. The main value of this study is the fact that it fills the gap between the conventional methods of statistical estimation and the intelligent methods of modern times. By giving a qualitative comparison of various estimation techniques, the study not only proves the superiority of SVM in the matter, but also the greater potential of hybrid statistical-computational techniques in the study of reliability. This article thus adds to the expanding body of literature supporting the incorporation of artificial intelligence into statistical modeling and provides practical implications of its use in engineering, biomedical sciences, and risk management. A number of avenues can be followed in future work. One of the steps to be followed is including Bayesian estimation of the inverse-Rayleigh NHPP. It would be possible to use Markov Chain Monte Carlo (MCMC) or variational inference to get the posterior distribution of the parameter α, which would allow the estimation uncertainty and credible intervals to be directly quantified and not just give point estimates. Such framework would allow to compare classical, machine learning, and Bayesian techniques on the levels of accuracy and uncertainty evaluation as a critical dimension of reliability application and risk-management practices. To begin with the analysis can be carried to other heavy-tailed or flexible lifetime distributions to assess whether the observed intelligence method superiority can be extended beyond the inverse Rayleigh situation. Second, hybrid structures that unite interpretability of classical methods with the forecasting significance of machine learning models might be created to improve estimation strength even more. Third, other more advanced deep learning architectures, like attention-based or convolutional neural networks, could be studied to bring even more improvements to understanding the complex temporal or structural patterns. Lastly, the integration of Bayesian learning methods may provide a probabilistic framework of parameter estimation, which allows the removal of the uncertainty as well as the point estimates. This paper highlights the relevance of the intelligent approach, especially the use of SVM in the models of reliability and forms the basis of new studies in the direction of the more progressive and hybrid estimation approaches.

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Ethical Approval

All the authors demonstrate that they have adhered to the accepted ethical standards of a genuine research study.

Competing Interests

No conflict of interest is declared by authors.

Author contributions

All authors have sufficiently contributed to the study and agreed with the results and conclusions.

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Availability of Data and Materials

The data used to support the findings of this study are included in the article.

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