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The $M/M/1/K_b$ Interdependent Queueing Model Includes Reverse Balking with Feedback and Controllable Arrival Rates

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Abstract
Objective: Some research in the queueing theory literature describes batch-delivery systems instead of only individualised one-on-one support. This study initially presents adjustable arrival percentages and interdependency in a system like these arrival and service procedures in order to determine the likelihood and characteristics of the system for queuing. It also validated the results that were attained.
Methods: With Poisson as the default assumption (that there is only one arrival for every Poisson occurrence), it is expected that the input will regulate the by varying arrival rates in speed and duration. The service is trustworthy and authorised for exact results. We are just using one server for the duration of this service. Re-entering the queue to enhance the outcome is what feedback does. Service is started when the number of customers approaches or beyond the capacity that is set aside. All of the probability in a stable state are found by a recursive technique.
Findings: Our utilisation feedback in $M/M/1/K_b$ model the properties and solutions of steady-state are determined and examined. The reneged customer and feedback customer, respectively, will have probabilities $p_1 and p_2$. Anticipated client volume and wait duration are contingent upon interdependencies, service frequency, rapid arrival frequency, and delayed arrival frequency. Based on every criteria, every outcome has been confirmed.
Novelty: There are some works related to a feedback queueing system, but this will is new approach of finding the best result for the required model with controllable arrival rates along with Mutual dependability of arrival and procedure for services.

Keywords Bivariate Poisson process, Interdependent Controllable Arrival and Service percentages, Limited Capacity, Reverse balking, One Server, feedback

AMS 2010 subject classifications 60K25, 68M20, 90B22

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1. Introduction

When a queue is long, it is common for arrivals to lose motivation and decide against joining it. This form of arrival is known called balking. A customer who may decide to return to the service repeatedly until the service is finished is referred to as feedback. In the writings of Haight[2], the concept of consumer balking occurs in queuing theory (1957). He conducted an evaluation of an infinitely long $M/M/1$ queue with baulking. In their study of "the $M/M/1/N$ queueing system using backward baulking", Jain and Rakesh Kumar [8] (2014) found that the likelihood of balking decreases as queue size increases. It is common practice to assume that the Arrival and service procedures operate separately, along with an amount of additional presumptions. The processes of arrival and services, however, are interdependent in many specific instances, therefore this must be taken into

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consideration. The term ”interdependent queuing model” refers to an orderly model where services are connected and arrivals are also connected. A lot of research possesses documented evidence among the research papers on the interdependent normative queuing model with adjustable arrival percentage. Shobha and K. S. Rao [4] (2000) have spoken about “$M/M/1/\infty$ interdependent queuing model with controllable arrival rates”. Dr. M Thiagarajan and A. Srinivasan [5] (2006) have discussed “the $M/M/1/K$ interdependent queueing model with controllable arrival rates”. Again Dr. M Thiagarajan and A. Srinivasan [6] (2007) had a discussion about “the $M/M/C/K/N$ interdependent queueing model with controllable arrival rates balking, reneging and spares”. Dr. M Thiagarajan and S. Sasikala (2016) have discussed “the $M/M/1/N$ interdependent queueing model with controllable arrival rates and reverse balking”. Yue, Rui, et al. [17] (2023) ”Capacity estimation at all-way stop-controlled intersections considering pedestrian crossing effects.” finds to help other values of controlled arrival rates. Yang, Yaoqi, et al. [18] (2023) ”Stochastic geometry-based age of information performance analysis for privacy preservation-oriented mobile crowdsensing” basic information about stochastic geometry. Mahanta, Snigdha, Nitin Kumar, and Gautam Choudhury [19] (2024) ”An analytical approach of Markov modulated Poisson input with feedback queue and repeated service under N-policy with setup time” makes a feedback on Markov modified Poisson analytic technique service . Xie, Qian, Li Jin and Jiayi Wang [20] (2023) ”Strategic Defense of Feedback-Controlled Parallel Queues against Reliability and Security Failures” enters strategic defence of controllable arrivals. Gayathri, S., and G. Rani [21] (2023) ”an $FM/M/C$ interdependent stochastic feedback arrival model of transient solution and busy period analysis with interdependent catastrophic effect” is a new method of analysis. Cherfaoui, Mouloud, Mohamed Boualem, and Amina Angelika Bouchentouf, [22] (2023) ”Modelling and simulation of Bernoulli feedback queue with general customers’ impatience under variant vacation policy” is an approach of Bernoulli response with overall annoyance customer who enters with vacation. Dr. M. Thiagarajan and A. Bebittovimalan (2024) [23] “An Infinite Capacity Single Server Markovian Queueing System With Discouraged Arrivals Retention Of Reneged Customers and Controllable Arrival Rates With Feedback” and Dr. M. Thiagarajan, A. Bebittovimalan (2024) [24] “A Finite Capacity Finite Source Single Server Markovian Queueing System with Discouraged Arrivals Retention of Reneged Customers and Controllable Arrival Rates with Feedback” are helpful to improvise this paper.

2. DESCRIPTION OF THE MODEL

λ_0 - *Faster rate of arrival*
λ_1 - *Slower rate of arrival*
μ - *Service rate*
ϵ - *Dependence rate*

1. Take a look at a system with just one server with limited capacity queuing, where clients come in a Poisson circulation with rates λ_0 and λ_1, and service durations exhibit exponential distribution with rate μ. The bivariate Poisson procedure with a combined Probability Mass Function (PMF) of the type is supposed to be followed by the procedure of arrival $(X_1(t))$ and the procedure of service $(X_2(t))$ off the system.

$$\mathcal{P}[X_1(t) = x_1,\ X_2(t) = x_2]$$
$$= e^{-(\lambda_i+\mu-\epsilon)t} \sum_{j=0}^{min(x_1\ x_2)} \frac{(\epsilon t)^j [(\lambda_i - \epsilon)t]^{x_1-j} [(\mu - \epsilon)t]^{x_2-j}}{j!(x_1 - j)!(x_2 - j)!}$$

where $x_1, x_2 = 0, 1, 2, ...\lambda_1 > 0, i = 0, 1; \mu > 0, 0 \leq \epsilon < min(\lambda_i, \mu), i = 0, 1$
2. The system has a limited capacity N
3. First Come First Serve is the rule for the line.
4. A consumer may decline with probability q_1 and enter with probability $p_1 = (1 - q_1)$ while the system is empty.

5. The likelihood that a client will quit the system is p_2, and the likelihood that they will become a feedback customer is q_2.
6. When the system has at least one client, the customer will baulk with the probability of $(1 - \frac{n}{(N_b-1)})$ and will thereafter come together with the probability of $\frac{n}{(N_b-1)}$.

3. THE EQUILIBRIUM STATE EQUATION

Let $P_n(0)$ indicate the chance of a stable state of having n consumers in the system with an arrival percentage of λ_0 , and $P_n(1)$ indicate the chance of a stable state with an arrival rate of λ_1 . It is noted that $P_n(0)$ is valid for $0 \leq n$ $leqr_b$, whereas each of them $P_n(0)$ and $P_n(1)$ are valid for $r_b + 1 \leq n \leq R_b - 1$. Furthermore, $P_n(1)$ is valid for $R_b \leq n \leq N_b$. Then $P_n(0) = P_n(1) = 0$ if $n > N_b$.

$$-(\lambda_0 - \epsilon)p_1\mathcal{P}_0(0) + p_2(\mu - \epsilon)\mathcal{P}_1(0) = 0 \tag{1}$$

$$-\left[\left(\frac{1}{N-1}\right)(\lambda_0 - \epsilon) + p_2(\mu - \epsilon)\right]\mathcal{P}_1(0) + p_2(\mu - \epsilon)\mathcal{P}_2(0) + (\lambda_0 - \epsilon)p_1(0)\mathcal{P}_0(0) = 0 \tag{2}$$

$$\begin{aligned} -\left[\left(\frac{n}{N-1}\right)(\lambda_0 - \epsilon) + p_2(\mu - \epsilon)\right]\mathcal{P}_n(0) + p_2(\mu - \epsilon)\mathcal{P}_{n+1}(0) \\ + \left(\frac{n-1}{N-1}\right)(\lambda_0 - \epsilon)\mathcal{P}_{n-1}(0) = 0 \quad n = 2, 3, ..., r_b - 1 \end{aligned} \tag{3}$$

$$\begin{aligned} -\left[\left(\frac{r_b}{N_b-1}\right)(\lambda_0 - \epsilon) + p_2(\mu - \epsilon)\right]\mathcal{P}_{r_b}(0) + p_2(\mu - \epsilon)\mathcal{P}_{r_b+1}(0) + p_2(\mu - \epsilon)\mathcal{P}_{r_b+1}(1) \\ + \left(\frac{r_b-1}{N_b-1}\right)(\lambda_0 - \epsilon)\mathcal{P}_{r_b-1}(0) = 0 \end{aligned} \tag{4}$$

$$\begin{aligned} -\left[\left(\frac{n}{N_b-1}\right)(\lambda_0 - \epsilon) + p_2(\mu - \epsilon)\right]\mathcal{P}_n(0) + p_2(\mu - \epsilon)\mathcal{P}_{n+1}(0) \\ + \left(\frac{n-1}{N_b-1}\right)(\lambda_0 - \epsilon)\mathcal{P}_{r_b-1}(0) = 0 \quad n = r_b + 1, r_b + 2, ..., R_b - 2 \end{aligned} \tag{5}$$

$$-\left[\left(\frac{R_b-1}{N_b-1}\right)(\lambda_0 - \epsilon) + p_2(\mu - \epsilon)\right]\mathcal{P}_{R_b-1}(0) + \left(\frac{R_b-2}{N_b-1}\right)(\lambda_0 - \epsilon)\mathcal{P}_{R_b-2}(0) = 0 \tag{6}$$

$$-\left[\left(\frac{r_b-1}{N_b-1}\right)(\lambda_1 - \epsilon) + p_2(\mu - \epsilon)\right]\mathcal{P}_{r_b+1}(1) + p_2(\mu - \epsilon)\mathcal{P}_{r_b+2}(1) = 0 \tag{7}$$

$$\begin{aligned} -\left[\left(\frac{n}{N_b-1}\right)(\lambda_1 - \epsilon) + p_2(\mu - \epsilon)\right]\mathcal{P}_n(1) + p_2(\mu - \epsilon)\mathcal{P}_{n+1}(1) \\ + \left(\frac{n-1}{N_b-1}\right)(\lambda_1 - \epsilon)\mathcal{P}_{r_b-1}(1) = 0 \quad n = r_b + 1, r_b + 2, ..., R_b - 1 \end{aligned} \tag{8}$$

$$\begin{aligned} -\left[\left(\frac{R_b}{N_b-1}\right)(\lambda_1 - \epsilon) + p_2(\mu - \epsilon)\right]\mathcal{P}_{R_b}(1) + p_2(\mu - \epsilon)\mathcal{P}_{R_b+1}(1) + \left(\frac{R_b-1}{N_b-1}\right)(\lambda_0 - \epsilon)\mathcal{P}_{R_b-1}(0) \\ + \left(\frac{R_b-1}{N_b-1}\right)(\lambda_1 - \epsilon)\mathcal{P}_{R_b-1}(1) = 0 \end{aligned} \tag{9}$$

$$-\left[\left(\frac{n}{N_b-1}\right)(\lambda_1-\epsilon)+p_2(\mu-\epsilon)\right]\mathcal{P}_n(1)+p_2(\mu-\epsilon)\mathcal{P}_{n+1}(1)$$
$$+\left(\frac{n-1}{N_b-1}\right)(\lambda_1-\epsilon)\mathcal{P}_{r_b-1}(1)=0 \quad n=R_b+1,R_b+2,...,N_b-1 \tag{10}$$

$$-p_2(\mu-\epsilon)\mathcal{P}_{N_b}(1)+\left(\frac{n-1}{N_b-1}\right)(\lambda_1-\epsilon)\mathcal{P}_{N_b-1}(1)=0 \tag{11}$$

From 1 we get,

$$\mathcal{P}_1(0)=\left[\frac{\lambda_0-\epsilon}{p_2(\mu-\epsilon)}\right]p_1\mathcal{P}_0(0)$$

Using the above result in 2, we get

$$\mathcal{P}_2(0)=\left[\frac{1}{N_b-1}\right]\left[\frac{\lambda_0-\epsilon}{p_2(\mu-\epsilon)}\right]^2 p_1\mathcal{P}_0(0)$$

Let$S_7=\dfrac{\lambda_0-\epsilon}{p_2(\mu-\epsilon)}$ and $T_7=\dfrac{\lambda_1-\epsilon}{p_2(\mu-\epsilon)}$

And hence we recursively derive

$$\mathcal{P}_n(0)=\left[\frac{(n-1)!}{(N_b-1)^{n-1}}\right](S_7)^n p_1\mathcal{P}_0(0), \quad n=1,2,...,r_b \tag{12}$$

$$\mathcal{P}_{r_b+1}(0)=\left[\frac{r_b!}{(N_b-1)^{r_b}}\right](S_7)^{r_b+1}p_1\mathcal{P}_0(0)-\mathcal{P}_{r_b+1}(1) \tag{13}$$

Using the above Result in 13 in 5, we recursively we derive

$$\begin{aligned}\mathcal{P}_n(0)&=\left(\frac{(n-1)!}{(N_b-1)^{n-1}}\right)(S_7)^n p_1\mathcal{P}_0(0)\\ &-\left[\left(\frac{S_7}{N_b-1}\right)^{n-r_b-1}(n-1)\mathcal{P}_{n-r_b-1}\right.\\ &+\left(\frac{S_7}{N_b-1}\right)^{n-r_b-2}(n-1)\mathcal{P}_{n-r_b-2}\\ &\left.+...+\left(\frac{S_7}{N_b-1}\right)^{n-R_b+1}(n-1)\mathcal{P}_{n-R_b+1}\right]\mathcal{P}_{r_b+1}(1)\\ &n=r_b+1,r_b+2,...,R_b-1\end{aligned} \tag{14}$$

Using the above result in 6, we get

$$\mathcal{P}_{r_b+1}(1)=\frac{\left(\frac{(R_b-1)!}{(N_b-1)^{R_b-1}}\right)(S_7)^{R_b}p_1\mathcal{P}_0(0)}{A_7} \tag{15}$$

Where

$$A_7=\left[\frac{(R_b-1)!}{r_b!}\left(\frac{S_7}{N_b-1}\right)^{R_b-r_b-1}+\frac{(R_b-1)!}{r_b+1!}\left(\frac{S_7}{N_b-1}\right)^{R_b-r_b-2}+...+\frac{(R_b-1)!}{(R_b-2)!}\left(\frac{S_7}{N_b-1}\right)+1\right]$$

From 7 we get

$$\mathcal{P}_{r_b+2}(1) = \left[\left(\frac{r_b+1}{N_b-1}\right)\left(\frac{\lambda_1-\epsilon}{p_2(\mu-\epsilon)}\right)+1\right]\mathcal{P}_{r_b+1}$$

Utilising the outcome mentioned above in 8,we obtain recursively

$$\begin{aligned} \mathcal{P}_n(1) = \Bigg[&\left(\frac{T_7}{N_b-1}\right)^{n-r_b-1}(n-1)\mathcal{P}_{n-r_b-1} \\ &+\left(\frac{T_7}{N_b-1}\right)^{n-r_b-2}(n-1)\mathcal{P}_{n-r_b-2} \\ +...+&\left(\frac{T_7}{N_b-1}\right)^{n-R_b}(n-1)\mathcal{P}_{n-R_b}\Bigg]\mathcal{P}_{r_b+1}(1) \\ &n = r_b+1, r_b+2, ..., R_b-1, R_b \end{aligned} \tag{16}$$

Using 16 in 9 we get

$$\mathcal{P}_{R_b+1}(1) = \left[\left(\frac{T_7}{N_b-1}\right)^{R_b-r_b}\frac{R_b!}{r_b!}+\left(\frac{T_7}{N_b-1}\right)^{R_b-r_b-1}\frac{R_b!}{(r_b+1)!}+...+\left(\frac{T_7}{N_b-1}\right)R_b\right]\mathcal{P}_{r_b+1}(1) \tag{17}$$

Utilising the outcome mentioned above in 10 and 11 we obtain recursively

$$\begin{aligned} \mathcal{P}_n(1) = \Bigg[&\left(\frac{T_7}{N_b-1}\right)^{n-r_b-1}(n-1)\mathcal{P}_{n-r_b-1} \\ &+\left(\frac{T_7}{N_b-1}\right)^{n-r_b-2}(n-1)\mathcal{P}_{n-r_b-2} \\ +...+&\left(\frac{T_7}{N_b-1}\right)^{n-R_b}(n-1)\mathcal{P}_{n-R_b}\Bigg]\mathcal{P}_{r_b+1}(1) \\ &n = R_b+1, R_b+2, ..., N_b-1, N_b \end{aligned} \tag{18}$$

Hence

$$\begin{aligned} \mathcal{P}_n(1) = \Bigg[&\left(\frac{T_7}{N_b-1}\right)^{n-r_b-1}(n-1)\mathcal{P}_{n-r_b-1} \\ &+\left(\frac{T_7}{N_b-1}\right)^{n-r_b-2}(n-1)\mathcal{P}_{n-r_b-2} \\ +...+&\left(\frac{T_7}{N_b-1}\right)^{n-R_b}(n-1)\mathcal{P}_{n-R_b}\Bigg]\mathcal{P}_{r_b+1}(1) \\ &n = r_b+1, r_b+2, ..., N_b-1, N_b \end{aligned} \tag{19}$$

12 to 19, we can find all the stable state probabilities are stated in therms of $\mathcal{P}_0(0)$.

4. PROPERTIES OF THE REQUIRED MODEL

The Likelihood $\mathcal{P}(0)$ that such a system is entering at a quicker rate is

$$\mathcal{P}(0) = \sum_{n=0}^{N_b} \mathcal{P}_n(0)$$

$$\mathcal{P}(0) = \sum_{n=0}^{r_b} \mathcal{P}_n(0) + \sum_{r_b+1=0}^{R_b-1} \mathcal{P}_n(0) + \sum_{n=R_b}^{N_b} \mathcal{P}_n(0)$$

$\mathcal{P}_n(0)$ occurs only when $n = 0 \leq r_b \leq R_b - 1$,

$$\mathcal{P}(0) = \mathcal{P}_0(0) + \sum_{n=1}^{r_b} \mathcal{P}_n(0) + \sum_{r_b+1=0}^{R_b-1} \mathcal{P}_n(0)$$

From 12,13 and 14 we get

$$\mathcal{P}(0) = \mathcal{P}_0(0) + \sum_{n=1}^{R_b-1} \left\{ \left(\frac{(n-1)!}{(N_b-1)^{n-1}} \right) S_7^n \right\} p_1 \mathcal{P}_0(0) - \left[\sum_{n=r_b+1}^{R_b-1} B_7 \right] D_7 \mathcal{P}_0(0) \tag{20}$$

where $B_7 = \left(\frac{S_7}{N_b-1} \right)^{n-r_b-1} (n-1)\mathcal{P}_{n-r_b-1} + \left(\frac{S_7}{N_b-1} \right)^{n-r_b-2} (n-1)\mathcal{P}_{n-r_b-2} + ... + \left(\frac{S_7}{N_b-1} \right)^{n-R_b+1} (n-1)\mathcal{P}_{n-R_b+1}$

Where $D_7 = \dfrac{\left(\frac{(R_b-1)!}{(N_b-1)^{R_b-1}} \right) (S_7)^{R_b} p_1}{A_7}$

Where $A_7 = \frac{(R_b-1)!}{r_b!} \left(\frac{S_8}{(N_b-1)} \right)^{R_b-r_b-1} + \frac{(R_b-1)!}{(r_b+1)!} \left(\frac{S_8}{(N_b-2)} \right)^{R_b-r_b-2} + ... + \frac{(R_b-1)!}{(R_b-2)!} \left(\frac{S_8}{(N_b-1)} \right) + 1$

The Likelihood $\mathcal{P}(0)$ that such a framework is entering at a slower rate is

$$\mathcal{P}(1) = \sum_{n=r_b+1}^{R_b} \mathcal{P}_n(1) + \sum_{n=R_b+1}^{N_b} \mathcal{P}_n(0) \mathcal{P}(1) = \sum_{n=r_b+1}^{N_b} \mathcal{P}_n(1)$$

From 19

$$\mathcal{P}(1) = \sum_{r_b+1=0}^{N_b} (C_7) D_7 \mathcal{P}_0(0) \tag{21}$$

Where $C_7 = \left(\frac{T_7}{N_b-1} \right)^{n-r_b-1} (n-1)\mathcal{P}_{n-r_b-1} + \left(\frac{T_7}{N_b-1} \right)^{n-r_b-2} (n-1)\mathcal{P}_{n-r_b-2} + ... + \left(\frac{T_7}{N_b-1} \right)^{n-R_b} (n-1)\mathcal{P}_{n-R_b}$

From the normalising condition, the probability $\mathcal{P}_0(0)$ that the framework is vacant may be computed.

$$\mathcal{P}(0) + \mathcal{P}(1) = 1$$

From 20 and 21 we get

$$\begin{aligned}\mathcal{P}(0) + \mathcal{P}(1) = \mathcal{P}_0(0) + \sum_{n=1}^{R_b-1} \left\{ \left(\frac{(n-1)!}{(N_b-1)^{n-1}} \right) S_7^n \right\} p_1 \mathcal{P}_0(0) \\ - \left[\sum_{n=r_b+1}^{R_b-1} B_7 \right] D_7 \mathcal{P}_0(0) \\ \sum_{n=r_b+1}^{N_b} (C_7)\, D_7 \mathcal{P}_0(0)\end{aligned} \tag{22}$$

$$\begin{aligned}\mathcal{P}_0(0) = \Bigg\{ 1 + \sum_{n=1}^{R_b-1} \left\{ \left(\frac{(n-1)!}{(N_b-1)^{n-1}} \right) S_7^n \right\} p_1 \\ - \left[\sum_{n=r_b+1}^{R_b-1} B_7 \right] D_7 + \sum_{r_b+1=0}^{N_b} (C_7)\, D_7 \Bigg\}^{-1}\end{aligned} \tag{23}$$

The system's typical customer count is provided by

$$L_s = L_{s_0} + L_{s_1}$$

$$L_{s_0} = \sum_{n=0}^{r_b} n\mathcal{P}_n(0) + \sum_{n=r_b+1}^{R_b-1} n\mathcal{P}_n(0) \tag{24}$$

$$L_{s_1} = \sum_{n=r_b+1}^{R_b-1} n\mathcal{P}_n(0) + \sum_{n=R_b}^{N_b} n\mathcal{P}_n(0) \tag{25}$$

From 12,13 and 14

$$\begin{aligned}L_{s_0} = \sum_{n=1}^{R_b-1} \left\{ \left(\frac{(n-1)!}{(N_b-1)^{n-1}} \right) S_7^n \right\} n p_1 \mathcal{P}_0(0) \\ - \left[\sum_{n=r_b+1}^{R_b-1} nB_7 \right] D_7 \mathcal{P}_0(0)\end{aligned} \tag{26}$$

From 15,16 and 17

$$L_{s_1} = \sum_{r_b+1=0}^{N_b} n\,(C_7)\, D_7 \mathcal{P}_0(0) \tag{27}$$

From 26 and 27 we get

$$\begin{aligned}L_s = \sum_{n=1}^{R_b-1} \left\{ \left(\frac{(n-1)!}{(N_b-1)^{n-1}} \right) S_7^n \right\} n p_1 \mathcal{P}_0(0) \\ - \left[\sum_{n=r_b+1}^{R_b-1} nB_7 \right] D_7 \mathcal{P}_0(0) \\ \sum_{r_b+1=0}^{N_b} n\,(C_7)\, D_7 \mathcal{P}_0(0)\end{aligned} \tag{28}$$

The anticipated wait time for users of the system is determined using Little's method as

$$W_s = \frac{L_s}{\overline{\lambda}} \tag{29}$$

where $\overline{\lambda} = \lambda_0 \mathcal{P}(0) + \lambda_1 \mathcal{P}(1)$

5. COMPUTATIONAL ILLUSTRATIONS

For fixed values of $r_b = 3, R_b = 6, N_b = 8, p_1 = 1 \& p_2 = 1$ and for different values of $\lambda_0, \lambda_1, \mu, \epsilon$ the virtues of $P_0(0), P(0), P(1), L_s$and W_s are calculated and compiled in the subsequent table.

Table 1. $\mathcal{P}(0), \mathcal{P}(1), L_s$ and W_s values

λ_1	μ	ϵ	ξ	$\mathcal{P}_{0,0}(0)$	$\mathcal{P}(0)$	$\mathcal{P}(1)$	L_s	W_s
4	3	4	0	0.5193	0.9474	0.0526	0.5776	0.1446
5	3	4	0	0.5190	0.9519	0.0481	0.5804	0.1451
6	3	4	0	0.5281	0.9672	0.0328	0.6218	0.1019
7	4	6	0.5	0.4524	0.9952	0.0048	0.7774	0.1962
7	5	6	0.5	0.4527	0.9932	0.0068	0.4743	0.1187
7	6	6	0.5	0.3522	0.9988	0.0012	0.4441	0.1180
8	4	5	0	0.5210	0.9732	0.0268	0.9763	0.2445
8	4	6	0	0.7257	0.9865	0.0135	0.7171	0.1799
8	4	7	0	0.6018	0.9972	0.0028	0.5224	0.1058
8	5	7	0.3	0.0710	0.9992	0.0008	1.0513	0.2664
8	5	7	0.6	0.6015	0.9981	0.0019	0.7046	0.1618
8	5	7	0.9	1.0082	0.9964	0.0036	0.6123	0.1239

6. GRAPHICAL REPRESENTATION

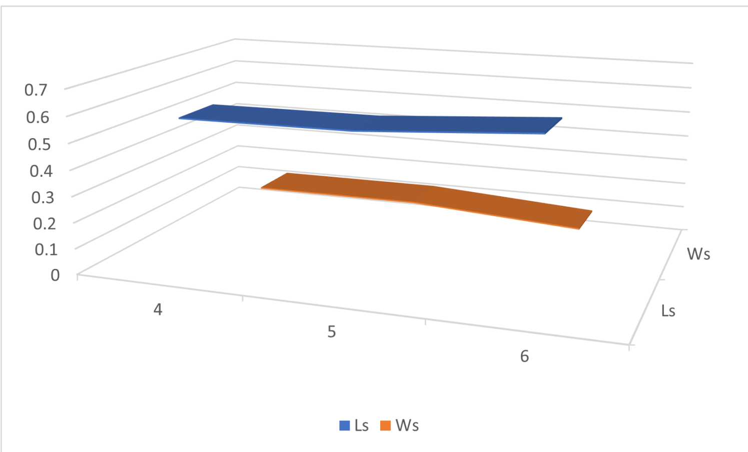

Figure 1. By varying the faster rate of arrival λ_0 while keeping other parameters fixed, we can determine how the mean quantity of users inside the network L_S together with the anticipated client wait time W_s are affected.

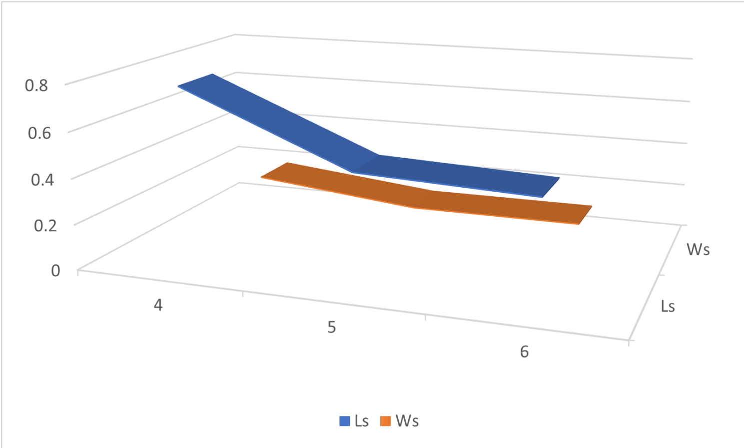

Figure 2. By varying the slower rate of arrival λ_1 while keeping other parameters fixed, we can determine how the mean quantity of users inside the network L_S together with the anticipated client wait time W_s are affected.

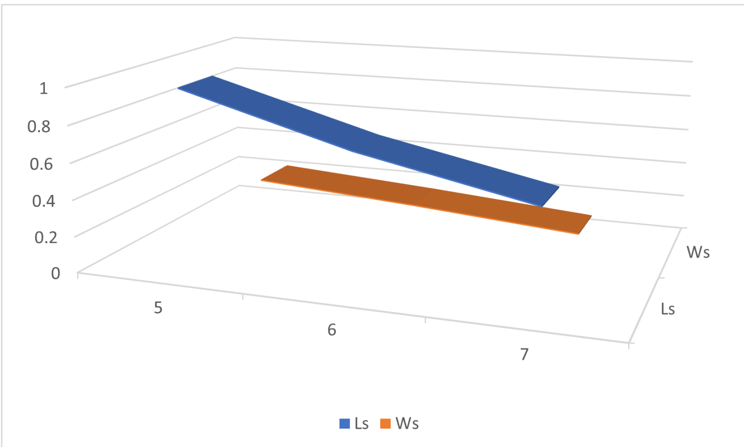

Figure 3. By varying the service rate of arrival μ while keeping other parameters fixed, we can determine how the mean quantity of users inside the network L_S together with the anticipated client wait time W_s are affected.

7. CONCLUSION

The findings based on the table consist of

- Once the mean dependency percentage increases while the additional factors remain continuous, L_s and W_s decline.
- L_s & W_s rise as the arrival rate rises while the other factors remain stable.
- L_s & W_s drop whenever the service percentage rises while the remaining factors stay identical. Additionally, it noticed that when q_1 is 1, the predicted framework size is 0.
- L_s climbs steadily and reaches its maximum when q_1 is zero when the balking rate is decreased while maintaining the other factors constant.
- W_s climbs steadily and reaches its maximum when q_2 is zero when the feedback rate is decreased while maintain the other factors constant.

The prior versions are included in this model as special situations. This exemplar simplifies to "$M/M/1/K$ correlated queueing exemplar with adjustable arrival rates, for instance, when $\xi = 0$[5]. When λ_0 tends to λ_1 and

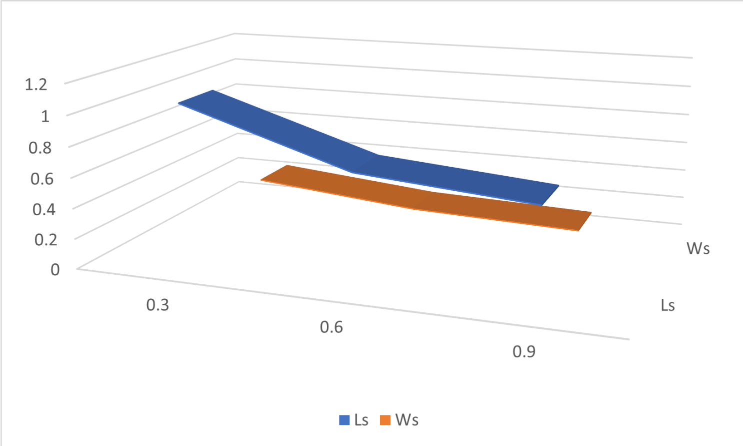

Figure 4. By varying the dependence rate of arrival ϵ while keeping other parameters fixed, we can determine how the mean quantity of users inside the network L_S together with the anticipated client wait time W_s are affected.

$\epsilon = 0$, this exemplar reduces to the "$M/M/1/K$ queueing model reverse balking with controllable arrival rates " [14].

The feedback queuing model enhances the ability to model and optimize systems where repeated service requests are likely. Consequently, clients receive their work quickly.The slower, faster, and idle system probabilities are computed. MATLAB is used to compute an average amount of clients and their wait times. Figures and tables are provided so that the customer flow may be easily understood.

In future, the present feedback queueing model can be extended by considering multiple servers or different service disciplines such as priority or bulk service. The model may also be enhanced by including customer behaviors like impatience, abandonment, or retrial. Introducing server breakdown and repair mechanisms would make the system more realistic.

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REFERENCES

1. Morse, P.M, Queues, Inventories and maintenances, Wiley, New York,(1958). Article DOI:https://doi.org/10.2307/2342909
2. Haight, F.A.,(1959) Queueing with reneging. Metrika 2; 186-196. Article DOI: http://dx.doi.org/10.1007/BF02613734
3. Ancker Jr.C.J. and Gafarian.A.V(1963). Some queueing problems with balking and reneging 1. Operations Research, 11;88-100. DOI: 10.1287/opre.11.1.88
4. Srinivasa Rao. K. Shobha. T, 2000, The M/M/1 interdependent queueing model with controllable arrival rates, Operation Research society of India. Vol.37,No.1. Article DOI: https://doi.org/10.1007/BF03398597
5. Srinivasan.A, and Thiagarajan.M, 2006, The M/M/1/K interdependent queueing model with controllable arrival rates. International journal of Management and systems Vol 22, No. 1, 23-34. Article DOI: https://doi.org/10.1007/s13571-011-0027-7
6. Srinivasan.A, and Thiagarajan.M, 2007, The M/M/C/K/N interdependent queueing model with controllable arrival rates balking reneging and spares, Journal of statistic and applications. Vol,2, No.1-2,56-65, Article DOI: https://doi.org/10.1007/s13571-011-0027-7
7. Kapodistria,S (2011). The M/M/1 Queue with Synchronized abandonments. Queueing system, 68,79-109. Article DOI: http://dx.doi.org/10.1007/s11134-011-9219-0
8. Kumar.R and Sharma,S.K.(2012) M/M/1/N Queueing system with retention of reneged customers. Pakistan Journal of Statistics and Operation Research, 8(4);859-866. Article DOI: https://doi.org/10.18187/pjsor.v8i4.408

9. Rakesh Kumar and Sumeet Kumar Sharma, A single server Markovian Queueing system with Discouraged Arrivals and Reneged Customers, Yogoslav Journal of operation Research 23(2013). No.2, DOI: http://dx.doi.org/10.2298/YJOR120911019K
10. Premalatha.S and Thiagarajan.M, A single server Markovian Queueing system with Discouraged Arrivals and Reneged Customers and controllable arrival rates, International Journal of Mathematical Archive 7(2),2016,129-134. Available from: https://www.ijsr.net/archive/v6i4/7041701.pdf
11. Medhi J. Stochastic models in queueing theory. and others, editor;Academic Press. 2002. Available from: https://www.elsevier.com/books/stochasticmodels-in-queueing-theory/medhi/978-0-12-487462-6
12. Shortle JF, Thompson JM, Gross D, Carl M Harris. Fundamentals of queueing theory. In: Fundamentals of queueing theory. John Wiley. 1974. Available from: https://doi.org/10.1002/9781119453765
13. Nithya, N., Anbazhagan, N., Amutha, S., Jeganathan, K., Park, G. C., Joshi, G. P., & Cho, W. (2023). Controlled arrivals on the retrial queueing–inventory system with an essential interruption and emergency vacationing server. *Mathematics*,11(16), 3560. Available from: https://doi.org/10.3390/math11163560
14. Sasikala, S., and Thiagarajan.M, "The M/M/1/N Interdependent Inter Arrival Queueing Model with Controllable Arrival Rates, Reverse Balking" *International Journal of Current Research*,Vol. 8, Issue, 01, pp.25311-25316, January, 2016 Available from:https://www.journalcra.com/sites/default/files/issue-pdf/12793.pdf
15. Sasikala, S., and V. Abinaya. "The M/M/C/N Interdependent Inter Arrival Queueing Model with Controllable Arrival Rates, Reverse Balking and Impatient Customers." *Arya Bhatta Journal of Mathematics and Informatics* 15.1 (2023): 1-10. Article DOI: http://dx.doi.org/10.5958/2394-9309.2023.00001.X
16. Sindhu, S., Achyutha Krishnamoorthy, and Dmitry Kozyrev. "On Queues with Working Vacation and Interdependence in Arrival and Service Processes." *Mathematics* 11.10 (2023): 2280. Available from: https://www.mdpi.com/2227-7390/11/10/2280
17. Yue, Rui, et al. "Capacity estimation at all-way stop-controlled intersections considering pedestrian crossing effects." *Transportation research record* 2677.3 (2023): 583-595. Article DOI: https://doi.org/10.1177/03611981221116368
18. Yang, Yaoqi, et al. "Stochastic geometry-based age of information performance analysis for privacy preservation-oriented mobile crowdsensing." *IEEE Transactions on Vehicular Technology* (2023). Article DOI: https://doi.org/10.1109/TVT.2023.3252167
19. Mahanta, Snigdha, Nitin Kumar, and Gautam Choudhury. "An analytical approach of Markov modulated Poisson input with feedback queue and repeated service under N-policy with setup time." *Quality Technology & Quantitative Management* 21.2 (2024): 257-285. Article DOI: https://doi.org/10.1080/16843703.2023.2193767
20. Xie, Qian, Jiayi Wang, and Li Jin. "Strategic Defense of Feedback-Controlled Parallel Queues against Reliability and Security Failures." *arXiv preprint arXiv:2301.11530* (2023). Article DOI: https://doi.org/10.48550/arXiv.2301.11530
21. Gayathri, S., and G. Rani. "AN FM/M/c INTERDEPENDENT STOCHASTIC FEEDBACK ARRIVAL MODEL OF TRANSIENT SOLUTION AND BUSY PERIOD ANALYSIS WITH INTERDEPENDENT CATASTROPHIC EFFECT." *Advances and Applications in Statistics* 86.1 (2023): 19-45. Article DOI: https://doi.org/10.17654/0972361723014
22. Cherfaoui, Mouloud, Amina Angelika Bouchentouf, and Mohamed Boualem. "Modelling and simulation of Bernoulli feedback queue with general customers' impatience under variant vacation policy." *International Journal of Operational Research* 46.4 (2023): 451-480. Article DOI: https://doi.org/10.1504/IJOR.2023.129959
23. Bebittovimalan A, Thiagarajan M. (2024) An Infinite Capacity Single Server Markovian Queueing System With Discouraged Arrivals Retention Of Reneged Customers and Controllable Arrival Rates With Feedback. *Indian Journal of Science and Technology*. 17(30):3100-3108. https://doi.org/10.17485/IJST/v17i30.1900
24. Bebittovimalan A, Thiagarajan M. (2024) A Finite Capacity Finite Source Single Server Markovian Queueing System with Discouraged Arrivals Retention of Reneged Customers and Controllable Arrival Rates with Feedback. *Indian Journal of Science and Technology*. 17(33): 3456-3466. https://doi.org/10.17485/IJST/v17i33.2354