STATISTICS, OPTIMIZATION AND INFORMATION COMPUTING
Stat., Optim. Inf. Comput., Vol. **15**, January 2026, pp 866–876.
Published online in International Academic Press (www.IAPress.org)

Two-Echelon Inventory Control Systems in Supply Chain with Fuzzy Number for Quadratic Demand

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Abstract In this paper, two-echelon inventory control system is discussed in supply chain using fuzzy number. The system consists of a manufacturer and single retailer. In our proposed model, manufacturer is an upper and retailer node is lower in echelon system and non-coordination supply chain model is investigated. Different types of demand models are given, particularly quadratic model is discussed. Here triangle fuzzy number is used for fuzzification and Graded Mean Integration and signed distance method are used for defuzzification. Some necessary theorems are proved.

Keywords Fuzzy set, α-cut of a fuzzy Set, Fuzzy number, Triangular fuzzy number, Defuzzification.

DOI: 10.19139/soic-2310-5070-2975

1. Introduction

A supply chain (SC) is an echelon structure which has different stages and each stage of the SC has its own motivation and different goals but they are dealt with same kind of things. No one has the likelihood of advancing whole SC execution. With respect to the situation, coordination and non-coordination are needed one in the supply chain management (SCM). Each member of SCM wishes to minimize the order quantity and maximize the profit. The Coordination plan prompts each member to adjust its goal to the entire SC.

In this paper non-coordination SC model is investigated. Fuzzy concept is applied in this model, it was more application and it has been giving an absolute accuracy in SC compare to crisp model. Here triangular fuzzy number is used for fuzzification; Graded Mean Integration and signed distance method are used to defuzzification. Different types of demand models are given, particularly linear, quadratic and exponential. The following papers were useful to work in this field and to initiate the paper.

In 1996, [13] Thomas, D.J. and Griffin, P.M. initiated the new model in SC that is Coordinated SCM. In the same year, 'Multi-echelon inventory models with fixed replenishment intervals', it was first initiated by Graves, S.C [4]. In 2005, "Fuzzy inventory (FI) with backorder defuzzification by signed distance method" it was discussed by [3] Chiang, J., et al. In 2007, Coordination in a single-vendor multi-buyer SC by synchronizing delivery and production cycles was dealt by Chan, C.K. and Kingsman, B.G. [2]. In the same year, [12] Syed J. K. and Aziz L. A. worked in this field of research "FIM without Shortages Using Signed Distance Method" and 'Coordination mechanisms of SC systems', it was given by Li, X. and Wang, Q [6].

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ISSN 2310-5070 (online) ISSN 2311-004X (print)

In 2010, three-stage supply chain coordination under production capacity bottleneck environment has been given by Sarmah, S.P [9]. In 2012, 'Coordination in two level SC with price dependent demand', it has been produced by [11] SyamSundar, K., et al. In this year, [7] Nagaraju, D., et al. initiated a new idea about 'Two-echelon SC with selling price dependent demand under wholesale price index and consumer price index'. In 2013, [8] Parvathi, P. and Gajalakshmi, S. gave an idea to know about the model 'A FIM with Lot Size Dependent Carrying Cost/ Holding Cost'. In the same year, [14] Uthayakumar, R. and Priyan, S. gave an idea in Pharmaceutical supply chain and inventory management strategies: Optimization for a pharmaceutical company and a hospital.

In 2014, Cooperative inventory model for vendor-buyer system with unequal-sized shipment, defective items and carbon emission cost has been investigated by [5] Jauhari, W.A., et al. In 2016, FIM with lot size dependent ordering cost in Healthcare Industries has been initiated by Uthayakumar,R. and Karuppasamy, S.K [15]. In 2016, three echelon SC with centralized and decentralized inventory decisions under linear price dependent demand was investigated by [1] Burra Karuna Kumar et al. Shan-Huo.C(1996) [10] investigated about the backorder FIM under function principle.

Justification for Fuzzy Approach: After the introduction of fuzzy set, the usage of fuzzy has been unpredicted growth in the world; what is the reason? That fuzzy approach is generalization of crisp concept, so fuzzification is applied in every field and the accuracy of the resulting data is better one; compare with existing data's. In this paper, we are using the fuzzy method to find the optimal value.

2. Preliminaries

Definition 2.1. Fuzzy Set. [15] A fuzzy set $\tilde{N}$ on the given universal set $\mathfrak{X}$ is a set of ordered pairs $\tilde{N} = \{(\mathfrak{x}, \mathfrak{F}_m(\mathfrak{x})) : \mathfrak{x} \in \mathfrak{X}\}$, where $\mathfrak{F}_m : \mathfrak{X} \to [0,1]$, is called membership function.

Definition 2.2. L-cut of $\mathfrak{A}$ Fuzzy Set. [15] The L-cut of a fuzzy set $\tilde{N}$, is defined by $\tilde{N}\mathfrak{L} = \{\mathfrak{x} : \mathfrak{F}_m(\mathfrak{x}) \geq L\}$, where $L \in [0,1]$.

Definition 2.3. Fuzzy Number. [15] A fuzzy subset of the real line R, whose membership function Ξ_m satisfies the following situation, is a generalized fuzzy number $\hat{N}$
(i) $\Xi_{\tilde{n}}$ is a continuous,
(ii) $\Xi_{\tilde{n}} = 0, -\infty < \mathfrak{t} \leq \mathfrak{q}_1$,
(iii) $\Xi_{\tilde{n}} = \mathfrak{T}(\mathfrak{t})$ is strictly increasing on $[\mathfrak{q}_1, \mathfrak{q}_2]$,
(iv) $\Xi_{\tilde{n}} = \mathfrak{U}_{\mathfrak{b}}, \mathfrak{q}_2 \leq \mathfrak{t} \leq \mathfrak{q}_3$,
(v) $\Xi_{\tilde{n}} = \mathfrak{R}(\mathfrak{t})$ is strictly decreasing on $[\mathfrak{q}_3, \mathfrak{q}_4]$,
(vi) $\Xi_{\tilde{n}} = 0, \mathfrak{q}_4 \leq \mathfrak{t} \leq \infty$, where $0 < \mathfrak{U}_{\mathfrak{b}} \leq 1$ and $\mathfrak{q}_1, \mathfrak{q}_2, \mathfrak{q}_3$ and $\mathfrak{q}_4 \in R$. It is denoted as $\tilde{N} = (\mathfrak{q}_1, \mathfrak{q}_2, \mathfrak{q}_3, \mathfrak{q}_4; \mathfrak{U}_{\mathfrak{b}})_{LR}$, if $\mathfrak{U}_{\mathfrak{b}} = 1$, then $\tilde{N} = (\mathfrak{q}_1, \mathfrak{q}_2, \mathfrak{q}_3, \mathfrak{q}_4)_{\mathfrak{T}\mathfrak{R}}$.

Definition 2.4. Triangular fuzzy number. [15] The fuzzy number $\tilde{N} = \{(\mathfrak{t}, \Xi_{\tilde{n}}(\mathfrak{t})) : \mathfrak{t} \in R\} = (\zeta_1, \zeta_2, \zeta_3)$ is called a triangular fuzzy number if

$$\Xi_{\tilde{n}} = \begin{cases} \frac{x-\zeta_1}{\zeta_2-\zeta_1}, & \zeta_1 \leq x \leq \zeta_2 \\ \frac{\zeta_3-x}{\zeta_3-\zeta_3}, & \zeta_2 \leq x \leq \zeta_3 \\ 0, & \text{otherwise} \end{cases}$$

Definition 2.5. The Function Principle. [10] Let $\tilde{N} = (\varsigma_1, \varsigma_2, \varsigma_3)$ and $\hat{U} = (\zeta_1, \zeta_2, \zeta_3)$ be two triangular fuzzy numbers. Then
(i) $\tilde{N} + \hat{U} = (\varsigma_1 + \zeta_1, \varsigma_2 + \zeta_2, \varsigma_3 + \zeta_3)$,
(ii) $\tilde{N} \times \hat{U} = (\varsigma_1\zeta_1, \varsigma_2\zeta_2, \varsigma_3\zeta_3)$, where $\varsigma_1, \varsigma_2, \varsigma_3, \zeta_1, \zeta_2, \zeta_3$ are all non zero positive real numbers,
(iii) $-\hat{U} = (-\zeta_3, -\zeta_2, -\zeta_1)$, and $\tilde{N} - \hat{U} = (\varsigma_1 - \zeta_3, \varsigma_2 - \zeta_2, \varsigma_3 - \zeta_1)$,
(iv) $\frac{1}{\hat{U}} = \hat{U}^{-1} = \left(\frac{1}{\zeta_3}, \frac{1}{\zeta_2}, \frac{1}{\zeta_1}\right)$, and $\frac{\tilde{N}}{\hat{U}} = \left(\frac{\varsigma_1}{\zeta_3}, \frac{\varsigma_2}{\zeta_2}, \frac{\varsigma_3}{\zeta_1}\right)$ where $\zeta_1, \zeta_2, \zeta_3$ are all non zero positive real numbers,.
(v) For any real number $\check{K}$, $\check{K}\tilde{N} = (\check{K}\varsigma_1, \check{K}\varsigma_2, \check{K}\varsigma_3)$ if $\check{K} > 0$, $\check{K}\tilde{N} = (\check{K}\varsigma_3, \check{K}\varsigma_2, \check{K}\varsigma_1)$ if $\check{K} < 0$.

Definition 2.6. Defuzzification. [15]
(i). Defuzzification of $\tilde{N} = (a_1, a_2, a_3)$ can be done by Graded Mean Integration Representation Method (GMIRM). It is defined as

$$G(\hat{N}) = \frac{1}{2}\frac{\int_0^1 \ell[a_1 + \ell(a_2 - a_1) + a_3 - \ell(a_3 - a_2)]d\ell}{\int_0^1 \ell d\ell} = \frac{a_1 + 4a_2 + a_3}{6}.$$

(ii). [3] The signed distance (SD) of $\tilde{N}$ is defined as

$$d(\hat{N}, 0) = S(\hat{N}) = \frac{1}{2}\int_0^1 [a_1 + \ell(a_2 - a_1) + a_3 - \ell(a_3 - a_2)]d\ell = \frac{a_1 + 2a_2 + a_3}{4}.$$

(iii). Total Integral Value Method (TIVM) of $\tilde{N}$ is defined as

$$TI(\hat{N}) = \int_0^1 [a_1 + \ell(a_2 - a_1) + a_3 - \ell(a_3 - a_2)]d\ell = \frac{a_1 + 2a_2 + a_3}{2}.$$

3. About the Model

3.1. Assumptions

The following are assumptions of this model;
(i) Fuzzy demand is expressed as a quadratic function of retailer's unit selling price.
(ii) Replenishment rate is instantaneous.
(iii) Replenishment batch size in the manufacturer is an integer multiple of replenishment quantity in the retailer.
(iv) Shortages are not allowed.
(v) Fuzzy Shipment quantity in each shipment to retailer from the manufacturer is same.

3.2. Notations

The following notations are used in this model;
X_R- Unit selling fuzzy price at the retailer.
D_R- The retailer annual fuzzy demand rate (units/year)=$A - BX_R - CX_R^2$ where $A, B, C > 0$.
T_C- Retailer cycle fuzzy time, expressed in terms of a year.
O_R- Fuzzy ordering cost of the retailer per cycle fuzzy time, T_C(per order).
S_M- Fuzzy setup cost of Manufacturer per setup for the cycle fuzzy time, ζT_C (per setup).
U_R- Fuzzy unit cost of the retailer (per unit).
U_M- Fuzzy unit cost of the manufacturer (per unit).
T_R- Retailer fixed fuzzy transportation cost to receive a consignment from the manufacturer (per shipment).
T_M- Manufacturer fixed fuzzy transportation cost for shipping a consignment to retailer (per shipment).
$\hat{S}_R$- Fuzzy shipment quantity in each shipment to replenish the inventory at the retailer from the manufacturer for the cycle fuzzy time, T_C (decision variable) ($\hat{S}_R = D_R T_C$).
ζ- Total number of fuzzy shipments to retailer from the manufacturer for the cycle fuzzy time, ζT_C (integer, decision variable).
$\hat{S}_M$- The manufacturer's replenishment batch fuzzy size/Production lot fuzzy size for the cycle fuzzy time, $\zeta T_C(\hat{S}_M = \zeta \hat{S}_R)$.
C_s- Fuzzy carrying charge (per Year).
$T_V(\hat{S}_R)$- The retailer's total variable fuzzy cost.
$T_V(\hat{S}_M, \zeta)$- The manufacturer total fuzzy variable cost.
$T(O)$- Total ordering fuzzy cost of the SC.
$T(C)$- Total carrying fuzzy cost of the SC.
$T(T)$- Total transportation fuzzy cost of the SC.
$T(V)$- The SC total variable fuzzy cost.

3.3. Non-Coordinated Supply Chain(NCSC)

In case of non-coordinated SC modeling, based on the fuzzy costs involved at his level, the retailer alone decides the optimal fuzzy ordering quantity, $\hat{S}_R$Depending on the quantity fuzzy ordered by the retailer and the various fuzzy costs involved at his level, manufacturer decides the most optimal number of shipments, ζ.

3.4. Total Variable Cost of the Retailer

The retailer's annual ordering ordering fuzzy cost $(R'sAOFC) = \frac{A-BX_R-CX_R^2}{\hat{S}_R}O_R$, where A and B are constants.
The retailer's annual ordering transportation fuzzy cost $(R'sATFC) = \frac{A-BX_R-CX_R^2}{\hat{S}_R}T_R$.
The retailer's annual ordering carrying fuzzy cost $(R'sACFC) = \frac{\hat{S}_R}{2}U_RC_s$.
The retailer's annual total variable fuzzy cost (T_V) is the sum of annual ordering fuzzy cost, transportation fuzzy cost and carrying fuzzy cost and is expressed as

$$T_V(\hat{S}_R) = \frac{A-BX_R-CX_R^2}{\hat{S}_R}O_R + \frac{A-BX_R-CX_R^2}{\hat{S}_R}T_R + \frac{\hat{S}_R}{2}U_RC_s.$$

Theorem 3.1. The annual total variable fuzzy cost of the retailer is convex in terms of $\hat{S}_R$. The optimal value of the replenishment fuzzy quantity $\hat{S}_R, \hat{S}_R^N$ is obtained by considering the first order and second order partial derivatives of the annual total variable fuzzy cost function with respect to $\hat{S}_R$ and is given by

$$\hat{S}_R = \sqrt{\frac{2(A-BX_R-CX_R^2)(O_R+T_R)}{U_RC_s}}.$$

Proof
The $R'sAT_VFC(T_V)$ is

$$T_V(\hat{S}_R) = \frac{A-BX_R-CX_R^2}{\hat{S}_R}O_R + \frac{A-BX_R-CX_R^2}{\hat{S}_R}T_R + \frac{\hat{S}_R}{2}U_RC_s.$$

Differentiating partially with respect to $\hat{S}_R$ to find optimal value,

$$\frac{\partial(T_V(\hat{S}_R))}{\partial\hat{S}_R} = -\frac{A-BX_R-CX_R^2}{(\hat{S}_R)^2}O_R - \frac{A-BX_R-CX_R^2}{(\hat{S}_R)^2}T_R + \frac{1}{2}U_RC_s,$$

again differentiating Partially with respect to $\hat{S}_R$, we get

$$\frac{\partial^2(T_V(\hat{S}_R))}{\partial\hat{S}_R^2} = 2\frac{A-BX_R-CX_R^2}{(\hat{S}_R)^2}O_R + 2\frac{A-BX_R-CX_R^2}{(\hat{S}_R)^3}T_R.$$

Since $\frac{\partial(T_V(\hat{S}_R))}{\partial\hat{S}_R} = 0, -\frac{A-BX_R-CX_R^2}{(\hat{S}_R)^2}O_R - \frac{A-BX_R-CX_R^2}{(\hat{S}_R)^2}T_R + \frac{1}{2}U_RC_s = 0$, which implies that

$$2(A-BX_R-CX_R^2)(O_R+T_R) = (\hat{S}_R)^2U_RC_s$$

$$\Rightarrow \hat{S}_R = \sqrt{\frac{2(A-BX_R-CX_R^2)(O_R+T_R)}{U_RC_s}} \tag{3.1.1}$$

And $\frac{\partial^2(T_V(\hat{S}_R))}{\partial\hat{S}_R^2} > 0$ for (3.1.1). Thus $\hat{S}_R = \sqrt{\frac{2(A-BX_R-CX_R^2)(O_R+T_R)}{U_RC_s}}$ is an optimal value. Hence $T_V(\hat{S}_R)$ is a convex.

Note 3.2. The principal minor of the Hessian matrix, $H(\hat{S}_R) = \frac{\partial^2(T_V(\hat{S}_R))}{\partial\hat{S}_R^2}$ for all values of and other model parameters. Thus, $\hat{S}_R$ happens to be the most optimal.

4. Total Variable Cost of the Manufacturer

The manufacturer's annual setup fuzzy cost $(M'sASFC) = \frac{A-BX_R-CX_R^2}{\zeta\hat{S}_R}S_M$.
The manufacturer's annual transportation fuzzy cost $(M'sATFC) = \frac{A-BX_R-CX_R^2}{\hat{S}_R}T_M$.
The manufacturer's annual carrying fuzzy cost $(M'sACFC) = \frac{(\zeta-1)\hat{S}_R}{2}U_M C_s$.
The manufacturer's annual total variable fuzzy cost (T_V) is the sum of annual setup fuzzy cost, transportation fuzzy cost and carrying fuzzy cost and is expressed as

$$\begin{aligned} T_V(\hat{S}_M,\zeta) &= \frac{A-BX_R-CX_R^2}{\zeta\hat{S}_R}S_M + \frac{A-BX_R-CX_R^2}{\hat{S}_R}T_M + \frac{(\zeta-1)\hat{S}_R}{2}U_M C_s \\ &= \frac{A-BX_R-CX_R^2}{\hat{S}_R}\left(\frac{S_M}{\zeta}+T_M\right) + \frac{(\zeta-1)\hat{S}_R}{2}U_M C_s. \end{aligned}$$

□

Theorem 4.1. For the given value of $\hat{S}_R$, the optimal fuzzy value of ζ, ζ^N constantly fulfils the following condition:

$$\zeta^N(\zeta^N-1) \le \frac{2(A-BX_R-CX_R^2)S_M}{U_M C_s \hat{S}_R^2} \le \zeta^N(\zeta^N+1).$$

Proof
For the given fuzzy value of $\hat{S}_R$, the optimal fuzzy value of ζ, ζ^N constantly fulfils the following conditions, $T_V(\hat{S}_M,\zeta^N) \le T_V(\hat{S}_M,\zeta^N-1)$ and $T_V(\hat{S}_M,\zeta^N) \le T_V(\hat{S}_M,\zeta^N+1)$.
The $M'sAT_VFC(T_V)$ is

$$T_V(\hat{S}_M,\zeta) = \frac{A-BX_R-CX_R^2}{\hat{S}_R}\left(\frac{S_M}{\zeta}+T_M\right) + \frac{(\zeta-1)\hat{S}_R}{2}U_M C_s \tag{4.1.1}$$

For, $T_V(\hat{S}_M,\zeta^N) \le T_V(\hat{S}_M,\zeta^N-1)$ (4.1.1) implies that

$$\begin{aligned} &\frac{A-BX_R-CX_R^2}{\hat{S}_R}\left(\frac{S_M}{\zeta^N}+T_M\right) + \frac{(\zeta^N-1)\hat{S}_R}{2}U_M C_s \le \frac{A-BX_R-CX_R^2}{\hat{S}_R}\left(\frac{S_M}{\zeta^N-1}+T_M\right) + \frac{(\zeta^N-2)\hat{S}_R}{2}U_M C_s \\ &\Rightarrow \frac{(A-BX_R-CX_R^2)S_M}{\hat{S}_R}\left(\frac{1}{\zeta^N}-\frac{1}{\zeta^N-1}\right) \le \frac{\hat{S}_R U_M C_s}{2}\left((\zeta^N-2)-(\zeta^N-1)\right) \\ &\Rightarrow \frac{(A-BX_R-CX_R^2)S_M}{\hat{S}_R}\left(\frac{-1}{\zeta^N(\zeta^N-1)}\right) \le \frac{\hat{S}_R U_M C_s}{2}(-1) \\ &\Rightarrow \frac{2(A-BX_R-CX_R^2)S_M}{\hat{S}_R^2 U_M C_s} \ge \zeta^N(\zeta^N-1) \end{aligned} \tag{4.1.2}$$

For, $T_V(\hat{S}_M,\zeta^N) \le T_V(\hat{S}_M,\zeta^N+1)$, (4.1.1) implies that

$$\begin{aligned} &\frac{A-BX_R-CX_R^2}{\hat{S}_R}\left(\frac{S_M}{\zeta^N}+T_M\right) + \frac{(\zeta^N-1)\hat{S}_R}{2}U_M C_s \le \frac{A-BX_R-CX_R^2}{\hat{S}_R}\left(\frac{S_M}{\zeta^N+1}+T_M\right) + \frac{\zeta^N\hat{S}_R}{2}U_M C_s \\ &\Rightarrow \frac{(A-BX_R-CX_R^2)S_M}{\hat{S}_R}\left(\frac{1}{\zeta^N}-\frac{1}{\zeta^N+1}\right) \le \frac{\hat{S}_R U_M C_s}{2}\left(\zeta^N-(\zeta^N-1)\right) \\ &\Rightarrow \frac{(A-BX_R-CX_R^2)S_M}{\hat{S}_R}\left(\frac{1}{\zeta^N(\zeta^N+1)}\right) \le \frac{\hat{S}_R U_M C_s}{2}(1) \\ &\Rightarrow \frac{2(A-BX_R-CX_R^2)S_M}{\hat{S}_R^2 U_M C_s} \le \zeta^N(\zeta^N+1) \end{aligned} \tag{4.1.3}$$

From (4.1.2) and (4.1.3), $\zeta^N(\zeta^N-1) \leq \frac{2(A-BX_R-CX_R^2)S_M}{U_M C_s \hat{S}_R^2} \leq \zeta^N(\zeta^N+1)$. □

Note 4.2. Subsequently, it is evident that the individually computed annual total variable fuzzy cost of the SC is equal to the sum of annual total variable fuzzy costs of manufacturer and retailer, i.e., $T(V) = T_V(\hat{S}_R) + T_V(\hat{S}_M, \zeta)$.

5. Calculations

Example 5.1. Fit the equation $D_R = A - BX_R - CX_R^2$ for the following relations of D_R and X_R

Selling price-X_R	3	4	5	6	7
Annual demand-D_R	5	10	15	5	10

by (i) crisp number (ii) fuzzy number.

Solution.
In Crisp, the normal equation of the equation $D_R = A - BX_R - CX_R^2$ are
$\Sigma D_R = nA - B\Sigma X_R - C\Sigma X_R^2$;
$\Sigma D_R X_R = A\Sigma X_R - B\Sigma X_R^2 - C\Sigma X_R^3$;
$\Sigma D_R X_R^2 = A\Sigma X_R^2 - B\Sigma X_R^3 - C\Sigma X_R^4$.
Here $n = 5$.

X_R	D_R	$X_R D_R$	X_R^2	X_R^3	X_R^4	$D_R X_R^2$
3	5	15	9	27	81	45
4	10	40	16	64	256	160
5	15	75	25	125	625	375
6	5	30	36	216	1296	180
7	10	70	49	343	2401	490
T=25	T=45	T=230	T=135	T=775	T=4659	T=1250

$45 = 5A - 25B - 135C$; $230 = 25A - 135B - 775C$;
$1250 = 135A - 775B - 4659C$, after solving, we get $A = -18.11$ and $B = -11.2$ & $C = 1.07$.
The equation is $D_R = -18.11 + 11.2X_R - 1.07X_R^2$
If $X_R = 4$ then $D_R = -18.11 + 11.2(4) - 1.07(16) = 9.57$
If $X_R = 5$ then

$$D_R = -18.11 + 11.2(5) - 1.07(25) = 11.14 \tag{5.1.1}$$

(ii) In fuzzy number, particularly triangle fuzzy number, the crisp numbers are fuzzified as

Fuzzy Selling price-X_R	(2,3,4)	(3,4,5)	(4,5,6)	(5,6,7)	(6,7,8)
Fuzzy annual demand-D_R	(4,5,6)	(9,10,11)	(14,15,16)	(4,5,6)	(9,10,11)

Then

X_R	D_R	$X_R D_R$	X_R^2	X_R^3	X_R^4	$D_R X_R^2$
(2,3,4)	(4,5,6)	(8,15,24)	(4,9,16)	(8,27,64)	(16,81,256)	(16,45,96)
(3,4,5)	(9,10,11)	(27,40,55)	(9,16,25)	(27,64,125)	(81,256,625)	(81,160,275)
(4,5,6)	(14,15,16)	(56,75,96)	(16,25,36)	(64,125,216)	(256,625,1296)	(224,375,576)
(5,6,7)	(4,5,6)	(20,30,42)	(25,36,49)	(125,216,343)	(625,1296,2401)	(100,180,294)
(6,7,8)	(9,10,11)	(54,70,88)	(36,49,64)	(216,343,,512)	(1296,2401,4096)	(324,490,704)
(20,25,30)	(40,45,50)	(165,230,305)	(90,135,190)	(440,775,1260)	(2274,4659,8674)	(745,1250,1945)

By the fuzzy normal equations, we get $(40,45,50)=5A-(20,25,30)B-(90,135,190)C$;
$(165,230,305)=(20,25,30)A-(90,135,190)B-(440,775,1260)C$;
$(745,1250,1945)=(90,135,190)A-(440,775,1260)B-(2274,4659,8674)C$, after solving, we get
$A=-(8.98,18.11,29.38)$, $B=-(9.06,11.2,13.34)$ and $C=(1.07,1.07,1.07)$.
Then $D_R=-(8.98,18.11,29.38)+(9.06,11.2,13.34)X_R-(1.07,1.07,1.07)X_R^2$.
If $X_R=(3,4,5)$, then

$$\begin{aligned}D_R&=-(8.98,18.11,29.38)+(9.06,11.2,13.34)(3,4,5)-(1.07,1.07,1.07)(9,16,25)\\&=-(8.98,18.11,29.38)+(27.18,44.8,66.7)-(9.63,17.12,26.75)\\&=(-8.55,9.57,27.69)\end{aligned}$$

By the GMIRM, defuzzification of D_R is $\frac{-8.55+4(9.57)+27.69}{6}=9.57$.
By the SDM, defuzzification of D_R is $\frac{-8.55+4(9.57)+27.69}{6}=9.57$.
If $X_R=(4,5,6)$ then

$$\begin{aligned}D_R&=-(8.98,18.11,29.38)+(9.06,11.2,13.34)(4,5,6)-(1.07,1.07,1.07)(16,25,36)\\&=-(8.98,18.11,29.38)+(36.24,56,80.04)-(17.12,26.75,38.52)\\&=(-31.66,11.34,53.94)\end{aligned}$$

By the GMIRM, defuzzification of D_R is

$$\frac{-31.66+4(11.34)+53.94}{6}=11.27 \tag{5.1.2}$$

By the SDM, defuzzification of D_R is $\frac{-31.66+2(11.34)+53.94}{4}=11.24$.
From (5.1.1) and (5.1.2), the fuzzy value is better than the crisp value.

Example 5.2. A television manufacturing company produces televisions. The company has a retailer. They are having the information; For retailer as selling price is Rs.300, the constants A=1,000,000, B=110 and C=10, ordering cost is Rs.300, unit cost of the item is Rs.600 and transportation cost to receive a consignment from the manufacturer is Rs.100. For Manufacturer as setup cost per order is Rs.1500,unit cost of an item is Rs.250 and Manufacturer fixed transportation cost for shipping is Rs.850.Also carrying charge per year 15% and $\zeta=2$.

Solution.
Given that **for Retailer**: X_R-selling price=Rs.300, A=1,000,000, B=110, and C=10
O_R-ordering cost per order=Rs.300,
U_R-Unit cost of the item=Rs.600,
T_R-transportation cost to receive a consignment from the manufacturer=Rs100.
T_C- Retailer cycle time, expressed in terms of a year=1.
And C_s-Carrying charge per year=15%.
Given that **for Manufacturer**: S_M-setup cost per order=Rs.1500,
U_M-Unit cost of an item=Rs.250,
T_M-Manufacturer fixed transportation cost for shipping=Rs.850.
In crisp case: For Retailer,
D_R=The retailer annual demand rate
$=A-BX_R-CX_R^2-CX_R^2$=1,000,000-110(300)-10(90,000)
=10,00,000-33,000-900,000=67000.
The optimal value of the replenishment quantity $\hat{S}_R=\sqrt{\frac{2(A-BX_R-CX_R^2)(O_R+T_R)}{U_RC_s}}$.
$\sqrt{\frac{2(6700)(300+100)}{600(0.15)}}=\sqrt{\frac{53600000}{90}}=\sqrt{595555.556}=771.72246$.
The retailer's annual ordering cost $=\frac{A-BX_R-CX_R^2-CX_R^2}{\hat{S}_R}O_R=\frac{67000}{771.72246}300=26045.633$.

The retailer's annual transportation cost $= \frac{A-BX_R-CX_R^2-CX_R^2}{\hat{S}_R}T_R = \frac{67000}{771.72246}100 = 8681.87768.$
The retailer's annual carrying cost $= \frac{\hat{S}_R}{2}U_RC_s = \frac{771.72246}{2}90 = 34727.5107.$
The retailer's annual total variable cost is
$T_V(\hat{S}_R) = \frac{A-BX_R-CX_R^2}{\hat{S}_R}O_R + \frac{A-BX_R-CX_R^2}{\hat{S}_R}T_R + \frac{\hat{S}_R}{2}U_RC_s$
=26045.633+8681.877+34727.5107=69455.0214
For Manufacturer,
The manufacturer's annual setup cost $= \frac{A-BX_R-CX_R^2}{\zeta\hat{S}_R}S_M = \frac{67000}{2(771.72246)}1500 = 65114.0826.$
The manufacturer's annual transportation cost is $\frac{A-BX_R-CX_R^2}{\hat{S}_R}T_M = \frac{67000}{771.72246}850 = 73795.9603.$
The manufacturer's annual carrying cost is $= \frac{(\zeta-1)\hat{S}^R}{2}U_MC_s = \frac{(2-1)771.72246}{2}250(0.15) = 14469.7961.$
The manufacturer's annual total variable cost (T_V) is
$T_V(\hat{S}_M,\zeta) = \frac{A-BX_R-CX_R^2}{\zeta\hat{S}_R}S_M + \frac{A-BX_R-CX_R^2}{\hat{S}_R}T_M + \frac{(\zeta-1)\hat{S}_R}{2}U_RC_s$
$= 65114.0826 + 73795.9603 + 14469.7961 = 153379.839.$
Annual total variable cost of the supply chain is $T(V) = T_V(\hat{S}_R) + T_V(\hat{S}_M,\zeta)$
$= 69455.0214 + 153379.839 = 222834.86$
In Fuzzy case: The crisp values are fuzzified by the triangle fuzzy number as For Retailer:
X_R-fuzzy selling price=Rs.(290, 300, 310), A=1,000,000, B=110, and C=10
O_R- fuzzy ordering cost per order=Rs.(290, 300, 310)
U_R- fuzzy Unit cost of the item=Rs.(590, 600, 610)
T_R- fuzzy transportation cost to receive a consignment from the manufacturer=Rs. (90,100,110).
T_C- Retailer cycle time, expressed in terms of a year=1.
And C_s-Carrying charge per year=15%.
For Manufacturer:
S_M- fuzzy setup cost per order=Rs.(1480,1500,1520)
U_M- fuzzy Unit cost of an item=Rs.(240, 250,260)
T_M- fuzzy Manufacturer fixed transportation cost for shipping=Rs.(840, 850, 860).
For Retailer, D_R-The retailer annual fuzzy demand rate=$A - BX_R - CX_R^2$
=1,000,000-110(290,300,310)-10(84100, 90000, 96100)
=1,000,000-(31900, 33000, 34100)-(841000, 900000, 961000)
=(965900,967000,968100)- (841000,900000,961000)
=(4900,67000,127100)
The optimal value of the replenishment quantity $\hat{S}_R = \sqrt{\frac{2(A-BX_R-CX_R^2)(O_R+T_R)}{U_RC_s}}$
$\sqrt{\frac{2(4900)(290+90)}{590(0.15)}} = \sqrt{\frac{3724000}{88.5}} = \sqrt{42079.096} = 205.131899,$
$\sqrt{\frac{2(67000)(300+100)}{600(0.15)}} = \sqrt{\frac{53600000}{90}} = \sqrt{595555.556} = 771.72246,$
$\sqrt{\frac{2(127100)(310+110)}{610(0.15)}} = \sqrt{\frac{106764000}{91.5}} = \sqrt{1166819.67} = 1080.19427$
By the Graded Mean Integration Representation Method, defuzzification of $\hat{S}_R$ is $\frac{205.131899+4(771.72246)+1080.19427}{6} = 728.702668.$
The retailer's annual ordering fuzzy cost $AOC_R = \frac{A-BX_R-CX_R^2}{\hat{S}_R}O_R$
$= \frac{4900}{205.131899}290 = 6927.25027, \frac{67000}{771.72246}300 = 26045.633, \frac{127100}{1080.19427}310 = 36475.8462.$
By the Graded Mean Integration Representation Method, defuzzification of AOC_R is

$$\frac{6927.25027 + 4(26045.633) + 36475.8462}{6} = 24597.6047.$$

The retailer's annual transportation fuzzy cost $AT_CR = \frac{A-BX_R-CX_R^2}{\hat{S}_R}T_R$
$\frac{4900}{205.131899}90 = 2149.83629, \frac{67000}{771.72246}100 = 8681.87768, \frac{127100}{1080.19427}110 = 12943.0421.$

By the Graded Mean Integration Representation Method, defuzzification of ATC_R is

$$\frac{2149.83629 + 4(8681.87768) + 12943.0421}{6} = 8303.39818.$$

The retailer's annual carrying fuzzy cost $ACC_R = \frac{\hat{S}_R}{2} U_R C_s$
$\frac{205.131899}{2} 88.5 = 9077.08658$, $\frac{771.72246}{2} 90 = 34727.5107$, $\frac{1080.19427}{2} 91.5 = 49418.8879$.
By the Graded Mean Integration Representation Method, defuzzification of ACC_R is $\frac{9077.08658+4(34727.5107+49418.8879)}{6} = 32901.0028$.
The retailer's annual total variable fuzzy cost is $T_V(\hat{S}_R) = \frac{A-BX_R-CX_R^2}{\hat{S}_R} O_R + \frac{A-BX_R-CX_R^2}{\hat{S}_R} T_R + \frac{\hat{S}_R}{2} U_R C_s$
$= (6927.25027, 26045.633, 36475.8462) + (2149.83629, 8681.87768, 12943.0421)$
$+(9077.08658, 34727.5107, 49418.8879) = (18154.1731, 69455.021, 98825.7763)$.
By the Graded Mean Integration Representation Method, defuzzification of $T_V(\hat{S}_R)$ is $\frac{18154.1731+4(69455.021)+98825.7763}{6} = 65802.0058$.

For Manufacturer,
The manufacturer's annual setup fuzzy cost $ASC_M = \frac{A-BX_R-CX_R^2}{\zeta \hat{S}_R} S_M$
$= \frac{4900}{2(205.131899)} 1480 = 17676.4317$, $\frac{67000}{2(771.72246)} 1500 = 65114.0826$, $\frac{127100}{2(1080.19427)} 1520 = 89424.6543$.
By the Graded Mean Integration Representation Method, defuzzification of $ASCM$ is $\frac{17676.4317+4(65114.0826)+89424.6543}{6} = 61259.5693$.
The manufacturer's annual transportation fuzzy cost $ATC_M = \frac{A-BX_R-CX_R^2}{\hat{S}_R} T_M$
$= \frac{4900}{205.131899} 840 = 20065.1387$, $\frac{67000}{771.72246} 850 = 73795.9603$, $\frac{127100}{1080.19427} 860 = 101191.057$.
By the Graded Mean Integration Representation Method, defuzzification of ATC_M is

$$\frac{20065.1387 + 4(73795.9603) + 101191.057}{6} = 69406.6728.$$

The manufacturer's annual carrying fuzzy cost $ACC_M = \frac{(\zeta-1)\hat{S}_R}{2} U_M C_s$
$\frac{(2-1)205.131899}{2} 240(0.15) = 3692.3742$, $\frac{(2-1)771.7224}{2} 250(0.15) = 14469.7961$,
$\frac{(2-1)1080.19427}{2} 260(0.15) = 21063.7883$
By the Graded Mean Integration Representation Method, defuzzification of ACC_M is

$$\frac{3692.3742 + 4(14469.7961) + 21063.7883}{6} = 13772.5578.$$

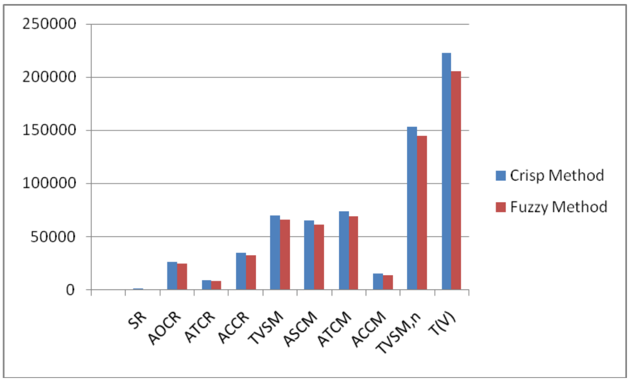
The manufacturer's annual total variable fuzzy cost (T_V) is
$T_V(\hat{S}_M, \zeta) = \frac{A-BX_R-CX_R^2}{\zeta \hat{S}_R} S_M + \frac{A-BX_R-CX_R^2}{\hat{S}_R} T_M + \frac{(\zeta-1)\hat{S}_R}{2} U_M C_s$
$= (17676.4317, 65114.0826, 89424.6543) + (20065.1387, 73795.9603, 101191.057)$
$+(3692.3742, 14469.7961, 21063.7883) = (41433.9446, 153379.839, 211679.499)$.
By the Graded Mean Integration Representation Method, defuzzification of $T_V(\hat{S}_M, \zeta)$ is

$$\frac{41433.9446 + 4(153379.839) + 211679.499}{6} = 144438.8.$$

Annual total variable fuzzy cost of the supply chain is $T(V) = T_V(\hat{S}_R) + T_V(\hat{S}_M, \zeta)$
$= (18154.1731, 69455.021, 98837.7763) + (41433.9446, 153379.839, 211679.411)$
$= (59588.1177, 222834.86, 210240.805)$.
By the Graded Mean Integration Representation Method, defuzzification of $T(V)$ is

$$\frac{59588.1177 + 4(222834.86) + 283059.823}{6} = 205664.563$$

S.No.		Crisp Method	Fuzzy Method
1	$\hat{S}_R$	771.72246	728.702668
2	AOC_R	26045.633	24597.6047
3	ATC_R	8681.87768	8303.39818
4	ACC_R	34727.5107	32901.0028
5	$T_V(\hat{S}_R)$	69455.0214	65802.0058
6	ASC_M	65114.0826	61259.5693
7	AT_CM	73795.9603	69406.6728
8	ACC_M	14469.7961	13772.5578
9	$T_V(\hat{S}_M, \zeta)$	153379.839	14438.8
10	T(V)	222834.86	210240.805



Analysis: From the graph, blue lines represent the optimum cost of manufacturer and retailer for the crisp data; Red lines represent the optimum cost of manufacturer and retailer for the fuzzy data; from these two lines,we can easily understand that fuzzy approached method is giving optimum cost.

6. Conclusion

In this work, two echelon supply chain is discussed, which consists of a manufacturer and retailer where the downstream retailer faces quadratic price dependent demand. A mathematical model for two echelon inventory system is developed under quadratic price dependent demand for non-coordinated supply chain. The proposed mathematical model can be useful to the industries dealing with fast moving consumer goods and durable goods where the demand is quadratic price sensitive. These industries can decide their inventory policies and shipment frequencies to optimize the total relevant cost. From the table, we can understand about the crisp model and fuzzy model, from these two models fuzzy model is best compare to the crisp model. The present work can be extended to coordinated supply chain with exponential demand, and other demand. Also the present work can be extended by considering the space cost, and advertisement cost with perishable products for analysing the multi-channel multi-echelon supply chain.

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