STATISTICS, OPTIMIZATION AND INFORMATION COMPUTING
Stat., Optim. Inf. Comput., Vol. **14**, November 2025, pp 0–15.
Published online in International Academic Press (www.IAPress.org)

Identifying Stability Criteria for Suggested Nonlinear Model with Application

Ammar Saad Abduljabbar, Anas S. Youns*, Salim M. Ahmad

Department of Mathematics, Mosul University, Iraq

Abstract This paper presents a study on forecasting the future population of the Arab Republic of Egypt using a proposed nonlinear autoregressive (NAR) model. The authors construct and train the model using historical census data from 1950 to 2023, aiming to project population trends from 2024 to 2033. The study focuses on identifying stability criteria for the proposed model and demonstrates its effectiveness in capturing the underlying patterns of Egypt's population dynamics. The results indicate a steady increase in population over the next decade, providing valuable insights for decision makers.

Keywords Ozaki Model; Stability Criteria; Prediction Models; Models for Regression.

AMS 2010 subject classifications 62M10, 62M20, 62M99, 62P99

DOI: 10.19139/soic-2310-5070-2657

1. Introduction

*W*e defined a suggested nonlinear autoregressive model and the definitions of the statistical criteria that we needed. The definitions of singular point with its stability in [1], stability of limit cycle, and the local linearization for the nonlinear model were found in [2]. The local linear approximation method was used to approximate the suggested model to a linear model; we also found the singular point and the stability conditions of it and the stability of the limit cycle if it exists for the suggested models of order one. We were given examples to explain stability for a singular point and the stability of the limit cycle of the suggested models of order one with the orbit plots for the example models by using Matlab programs. Also, we used the suggested model to predict the population of Egypt for the next twenty years after 2023 after fulfilling the stability conditions and achieving the statistical criteria of the suggested model of order one. After that, the conclusions of the research were mentioned. At the end of the research, it closed the suggestions for future work. Numerous researchers have conducted many studies similar to ours. The researchers "Amal I. Saba and Ammar H. Elsheikh "predicted the spread of the COVID-19 outbreak in Egypt using nonlinear autoregressive artificial neural networks in [3]. "Ban Ghanim Al-Ani "studied the statistical analysis of the novel COVID-19 epidemic in Iraq in [4]. "Qais M. Abdulqader" predicted the ratio of the rural population in Iraq using the Box-Jenkins methodology in [5]. "Mohamed R. Abonazel and Nesma M. Darwish "conducted predictions of COVID-19 cases in Egypt using the ARIMA Box-Jenkins method in [6]. "Abdelrahim M. Zabadi and colleagues" conducted a study with the goal of developing a mathematical approach to forecast Jordan's population up to the year 2100 in [7]. "Roberto Baragona and his team" examined the estimation method proposed by Hagan and Ozaki, which employs a regression time series model to represent nonlinear stochastic oscillations [8]. "Ayoub Ahmed and his team" analyzed a model of the coronavirus disease (COVID-19) using numerical analysis techniques and a logistic model in [9]. "Anis A. Khadoum and his team" used the regression

*Correspondence to: Anas S. Youns (Email: anass.youns@uomosul.edu.iq). Department of Mathematics, Mosul University, Mosul, Nineveh, Iraq (41002).

ISSN 2310-5070 (online) ISSN 2311-004X (print)

method to analyze the spread of the new coronavirus (COVID-19) in Iraq, relying on mathematical and dynamic neural network models in [10]. "Chandrakanta Mahanty et al." used two nonlinear growth models (Gompertz and Verhulst) as well as the exponential model (SIR) to analyze the global coronavirus pandemic in [11]. "Rabiu Aliyu Abdulkadir and his colleagues "used a comparison between linear and nonlinear models for daily rainfall forecasts at Ercan Airport, North Cyprus, in [12]. "Minglu Ma and other researchers" studied linear and nonlinear composite models applied to forecasting coal consumption in South Africa in [13]. Anas S. Youns and Salim M. Ahmad suggested "Stability Conditions for a Nonlinear Time Series Model" in [14]. Salim M. Ahmad, Anas S. Youns, and Manal S. Hamdi proposed "Nonlinear Autoregressive Model for Stability and Prediction" in [15]. Moatasem Yaseen Al-Ridha, Ammar Sameer Anaz, and Raid Rafi Omar Al-Nima proposed "Expecting confirmed and death cases of covid-19 in Iraq by utilizing backpropagation neural network" in [16]. Osamah Basheer Shukura, Sabah Hussein Alib, and Layali Adil Saberc, "Forecasting climate temperature data in Nineveh Governorate using the recurrent neural network method," in [17]. Osamah Basheer Shukur and Muhammad Hisyam Lee studied "Daily wind speed forecasting using ARIMA-based hybrid KF-ANN model" in [18].

2. The Suggest Model and the Statistical Criteria:

The nonlinear suggest time series model is that

$$w_t = \sum_{i=1}^{p} \left[c_i + d_i \left(\frac{1}{-w_{t-1}+1} \right) \right] w_{t-i} + \varepsilon_t \quad ; \quad w_{t-1} \neq -1 \tag{1}$$

Where: c_1,...,c_p,d_1,...,d_p indicate constants and $\{\varepsilon_t\}$ is random noise.
The statistical criteria are that:
The mean square error is

$$\text{MSE} = \frac{1}{n}\sum_{i=1}^{n}(w_i - \hat{w}_i)^2$$

w_i denoted by the real data and $\hat{w}_i$ the predicted data, as described in [19].
The Root Mean Square Error (RMSE) is defined as:

$$\text{RMSE} = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(w_i - \hat{w}_i)^2}$$

where w_i the real data and $\hat{w}_i$ the predicted data are described in [20].
The mean absolute error (MAE) is given by:

$$\text{MAE} = \frac{1}{n}\sum_{i=1}^{n}|w_i - \hat{w}_i|$$

where w_i are the real data, and $\hat{w}_i$ are the predicted data described in [21].
The Mean Absolute Percentage Error (MAPE) is defined as:

$$\text{MAPE} = \frac{1}{n}\sum_{i=1}^{n}\left|\frac{w_i - \hat{w}_i}{w_i}\right|$$

where w_i the real data and $\hat{w}_i$ are the predicted data described in [22].
The coefficient of determination (R^2) is computed as:

$$R^2 = 1 - \frac{\sum_{i=1}^{n}(w_i - \hat{w}_i)^2}{\sum_{i=1}^{n}(w_i - \bar{w})^2}$$

where w_i are the real data, $\hat{w}_i$ are the predicted data, and $\bar{w}$ is the mean of w_i described in [23].

The residual variance (based on Maximum Likelihood Estimation) is defined as

$$\sigma^2 = \frac{1}{n-k}\sum_{i=1}^{n}(w_i - \hat{w}_i)^2$$

where w_i are the real data, and $\hat{w}_i$ are the predicted data described in [24].

Moreover, the definitions of the statistical criteria Akaike's Information Criterion (AIC), Bayesian Information Criterion (BIC), and Normalized BIC (NBIC) can be found in [23]. These criteria are defined as follows:

$$\text{AIC} = -2 \cdot \log(\sigma^2) + 2k$$
$$\text{BIC} = -2 \cdot \log(\sigma^2) + k \cdot \log(n)$$
$$\text{NBIC} = \frac{BIC}{n}$$

where: n: number of observations, k: number of estimated parameters, σ^2: residual variance.

3. The Results

3.1. Identify Z (singular point):

Consider the proposed first-order model, such as

$$w_t = \left[c_1 + d_1\left(\frac{1}{-w_{t-1}+1}\right)\right] w_{t-1} + \varepsilon_t \quad ; \quad w_{t-1} \neq 1 \tag{2}$$

Hence$Z = f(Z)$, the random noise $\varepsilon_t = 0$ in (2).

Then $Z = \left[c_1 + d_1\left(\frac{1}{-Z+1}\right)\right] Z$

Since, $Z \neq 0$, $Z \neq 1$,$d_1 \neq 0$ and $c_1 \neq 1$.

Therefore,

$$Z = 1 - (\frac{d_1}{1-c_1}) \tag{3}$$

Where, $1 - c_1 \neq 0$,$d_1 \neq 0$

The suggested model of order two

$$w_t = \left[c_1 + d_1\left(\frac{1}{-w_{t-1}+1}\right)\right] w_{t-1} + \left[c_2 + d_2\left(\frac{1}{-w_{t-1}+1}\right)\right] w_{t-2} + \varepsilon_t \quad ; \quad w_{t-1} \neq 1 \tag{4}$$

Whenever, $\varepsilon_t = 0$, $Z = f(Z)$,

Therefore, $Z = \left[c_1 + d_1\left(\frac{1}{-Z+1}\right)\right] Z + \left[c_2 + d_2\left(\frac{1}{-Z+1}\right)\right] Z$

The singular point Z for (4) is

$$Z = 1 - (\frac{\sum_{i=1}^{2} d_i}{1 - (\sum_{i=1}^{2} c_i)}) \tag{5}$$

Where, $(1 - (\sum_{i=1}^{2} c_i)) \neq 0$; $(\sum_{i=1}^{2} d_i) \neq 0$

Then Z for equation (1) is determined using a technique similar to that used for equations (3) and (5), such that

$$Z = 1 - (\frac{\sum_{i=1}^{p} d_i}{(1 - \sum_{i=1}^{p} c_i)}) \tag{6}$$

Where, $(1 - (\sum_{i=1}^{p} c_i)) \neq 0$; $(\sum_{i=1}^{p} d_i) \neq 0$

3.2. *Singular point stability:*

Put $w_s = Z + Z_s$, for all $s = t; t-1$, $\varepsilon_t = 0$ in (2).

Since Z_s;$s = t; t-1$ is exceedingly small, then $Z_t.Z_{t-1} = 0$.

Also , $\forall s = t; t-1$,$\forall n \geq 2$, Z_s^n convergent to zero.

Hence, by applying the Taylor series expansion in (2). Therefore

$$Z + Z_t = [c_1 + d_1(\frac{1}{-(Z + Z_{t-1}) + 1})](Z + Z_{t-1}) \tag{7}$$

To obtain

$$Z_t = [\frac{Z - 2c_1 Z + c_1 + d_1}{(-Z + 1)}]Z_{t-1} \tag{8}$$

Therefore

$$Z_t = a_1 Z_{t-1}; a_1 = [\frac{Z - 2c_1 Z + c_1 + d_1}{(-Z + 1)}] \tag{9}$$

Therefore, if the root of equation (8) lies in the unit circle, equation (9) represents a stable first-order model $\mid r_1 \mid = \mid a_1 \mid < 1$.

3.3. *The limit cycle*

A period q limit cycle of $w_t = w_t;\ w_{t+1}; \ldots; w_{t+q}$, for the suggested model in the equation (2).When w_s is a points nearly a limit cycle is replaced for all $s = t; t-1$, $w_s = w_s + Z_s$, $\varepsilon_t = 0$

$$w_t + Z_t = [c_1 + d_1(\frac{1}{-(w_{t-1} + Z_{t-1}) + 1})](w_{t-1} + Z_{t-1}) \tag{10}$$

Then

$$Z_t = \left[\frac{w_t - 2c_1 w_{t-1} + c_1 + d_1}{(-w_{t-1} + 1)}\right] Z_{t-1} \tag{11}$$

Let t=t+q in (11)

$$Z_{t+q} = \left[\frac{w_{t+q} - 2c_1 w_{t+q-1} + c_1 + d_1}{(-w_{t+q-1} + 1)}\right] Z_{t+q-1} \tag{12}$$

Therefore

$$Z_{t+q} = \prod_{i=1}^{q} \left[\frac{w_{t+q-(i-1)} - 2c_1 w_{t+q-i} + c_1 + d_1}{-w_{t+q-i} + 1}\right] Z_t \tag{13}$$

Then

$$\left|\frac{Z_{t+q}}{Z_t}\right| = \left|\prod_{i=1}^{q} \frac{w_{t+q-(i-1)} - 2c_1 w_{t+q-i} + c_1 + d_1}{(-w_{t+q-i} + 1)}\right| < 1 \tag{14}$$

Then, (14) is

$$\left|\frac{Z_{t+q}}{Z_t}\right| = \left|\prod_{i=1}^{q} \frac{w_{t+i} - 2c_1 w_{t+i-1} + c_1 + d_1}{(-w_{t+i-1} + 1)}\right| < 1 \tag{15}$$

4. Numerical Examples

The examples in this section display the procedure of single point determination for the proposed first-order model and verifying the stability condition of this point. In example 1, the singular point was determined, and the stability condition was met. Using MATLAB, the model trajectories were plotted for various initial values, revealing that the system stabilizes at the singular point, as depicted in figure 1. Consequently, the model is orbitally stable. In example 2, the singular point was identified, but the stability condition was not fulfilled. MATLAB was again used to plot the trajectories for different initial values, showing that the system fails to stabilize at the singular point, as illustrated in figure 2. Thus, the model is orbitally unstable.

Example (1):

If $c_1 = 0.9$, $d_1 = 0.2$, $\varepsilon_t = 0$, in (2) to reach

$w_t = \left[0.9 + 0.2\left(\frac{1}{-w_{t-1}+1}\right)\right] w_{t-1}; \quad w_{t-1} \neq 1$

Then, by using equation (3), we reach that $Z = 1 - \frac{d_1}{1-c_1} = 1 - \frac{0.2}{1-0.9} = 1 - \frac{0.2}{0.1} = 1 - 2 = -1$

Hence, by used (9), then $Z_t = 0.95 Z_{t-1}$ for $Z = -1$.

Since the root of the previous equation lies inside the unit circle, the singular (stability) point $Z = -1$ in this example is confirmed to be stable. Figure 1 below illustrates a stability for a model across different initial values.

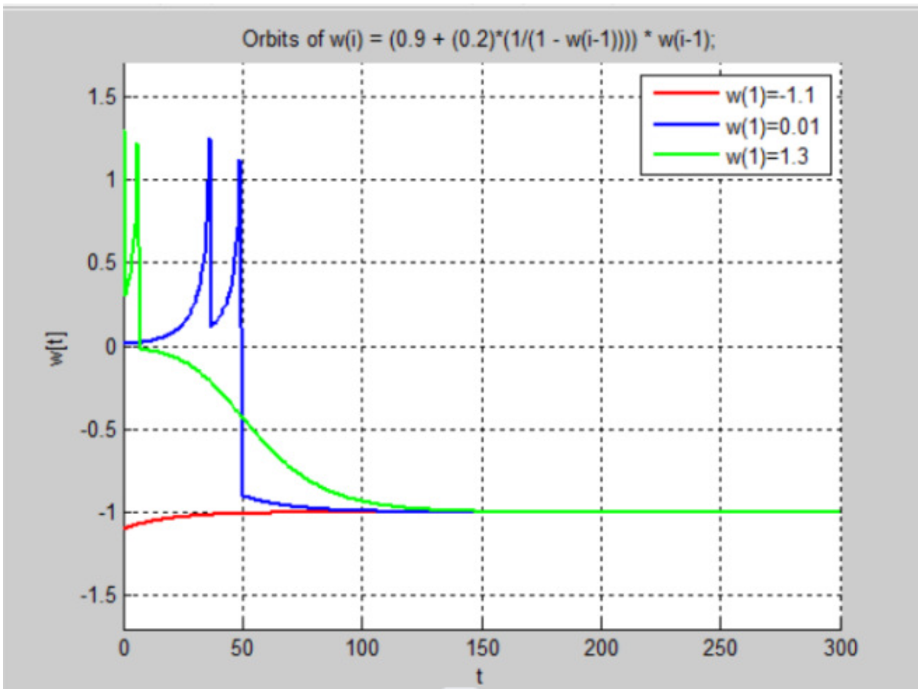


Figure 1. Z = -1 is a stable singular point with various initial values.

Example (2):

Whenever, $c_1 = -0.06$, $d_1 = -1.01$, $\varepsilon_t = 0$, in (2) to reach

$w_t = \left[-0.06 + (-1.01)\left(\frac{1}{-w_{t-1}+1}\right)\right] w_{t-1}; \quad w_{t-1} \neq 1.$

Then, by using an equation (3) to get that $Z = 1 - \frac{d_1}{1-c_1} = 1 - \frac{-1.01}{1+0.06} = 1 + \frac{1.01}{1.06} = 1.953$

Since,$Z = 1.953$, and applied (9), to get $Z_t = -1.1725 Z_{t-1}$

Given that the previous equation's root is located outside the unit circle, then Z is determined to be unstable. Figure (2) below illustrates the model's instability for various initial values.

5. The data analysis for an application

5.1. Description of the data used for the study

The research made use of data obtained from the website of the population of Egypt (https://www.populationpyramid.net/Egypt/1950/), which provides information on the population of Egypt for the

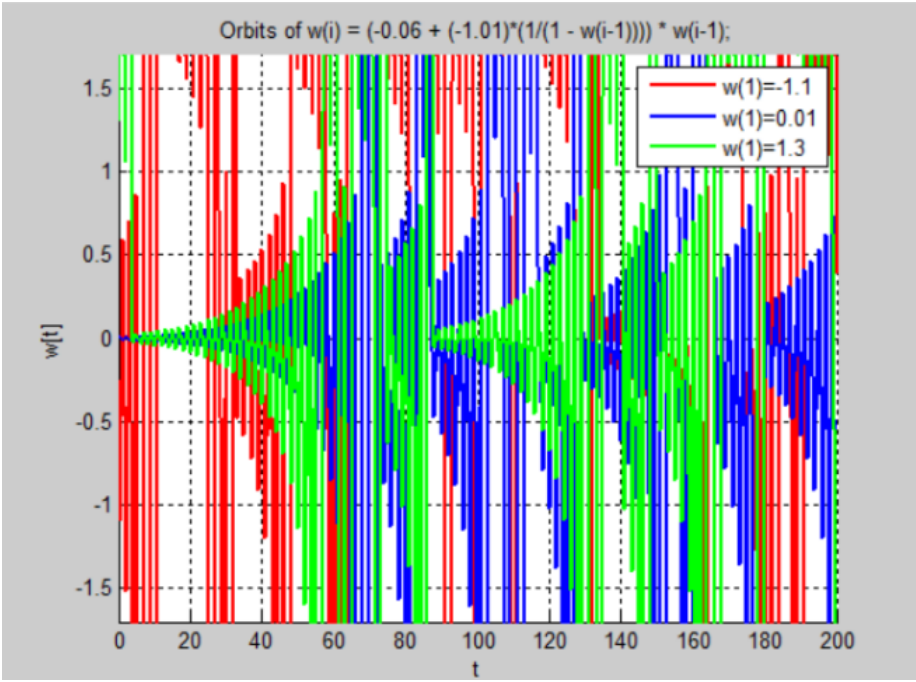


Figure 2. Z=1.953 is not a stable singular point with various initial values.

years 1950 to 2023. The dataset spans 74 years. The lowest population count in Egypt was recorded in 1950 at 21,150,058 individuals, while the highest population count occurred in 2023 at 112,716,598 individuals. For access to the data utilized in the study, please consult Appendix(A). The next figure, figure (3), displays the POP = Wt series data between 1950 and 2023.

By utilizing the SPSS program to analyze the graph of the primary POP data series as shown in Figure (3), it becomes evident that the data is exhibiting growth, following an exponential pattern, and displaying instability. Through the utilization of the Eviews9 software, an analysis was conducted on the initial POP dataset (Wt) using autocorrelation and partial autocorrelation functions, as depicted in Figure (4). The observation made from this analysis was that the POP variable lacks stability.

Due to the exponential nature of the original series, a fresh series named LPOP was generated by logarithmically transforming the POP data. Subsequently, autocorrelation and partial autocorrelation functions were graphed for the LPOP series. Nonetheless, upon observation, the newly derived LPOP time series data was found to exhibit instability, as illustrated in Figure (5).

After applying the first difference to the original data by taking the natural logarithm, we obtained a new variable $\text{DLPOP} = \log(W_t) - \log(W_{t-1}) = Z_t$. The autocorrelation and partial autocorrelation functions were plotted for this new variable DLPOP, as shown in Figure (6).

The Figure (6) displays the resulting stable time series and can be used for predicting future observations. The following figure, Figure (7), shows the series is stable of logarithms of the first difference of the original data that $\text{DLPOP} = Z_t$.

5.2. Estimate the parameters and forecasted and prediction for the study model with data

Python program were employed to determine optimal parameter values for the study data and the proposed non-linear model, as well as to ensure stability conditions as outlined in the third paragraph of the research. The link provided directs to the Python program: https://colab.research.google.com/drive/1ftG-Rwh28y3yGGLHCiBfb90odjeZK_LB#scrollTo=RVpBFxJKzyXS

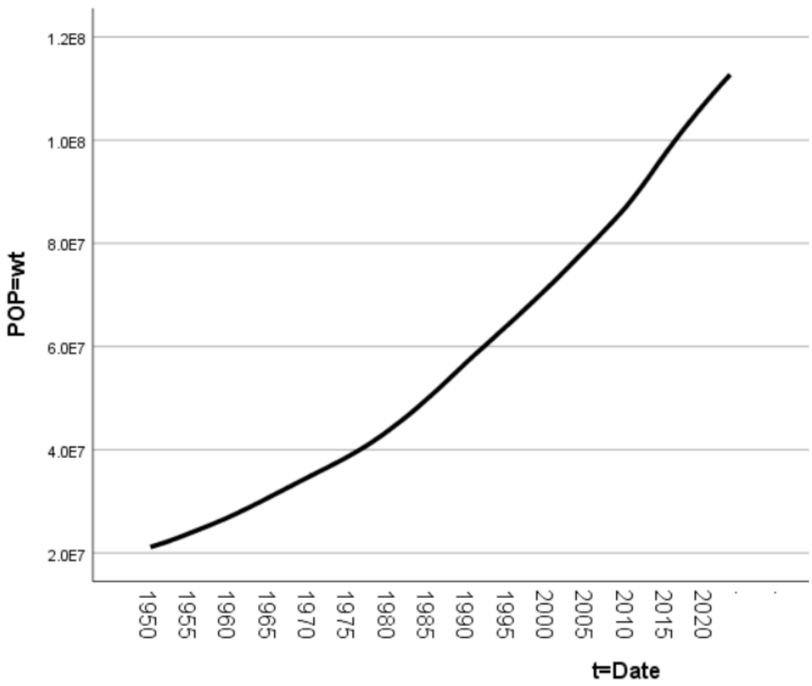
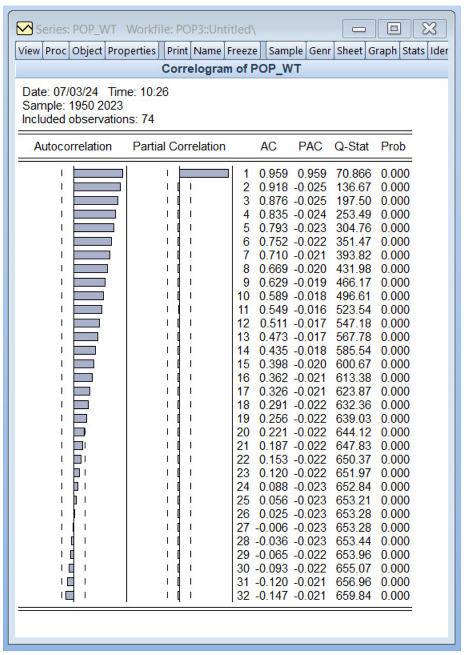


Figure 3. The POP series data between 1950 and 2023.


Series: POP_WT Workfile: POP3::Untitled\

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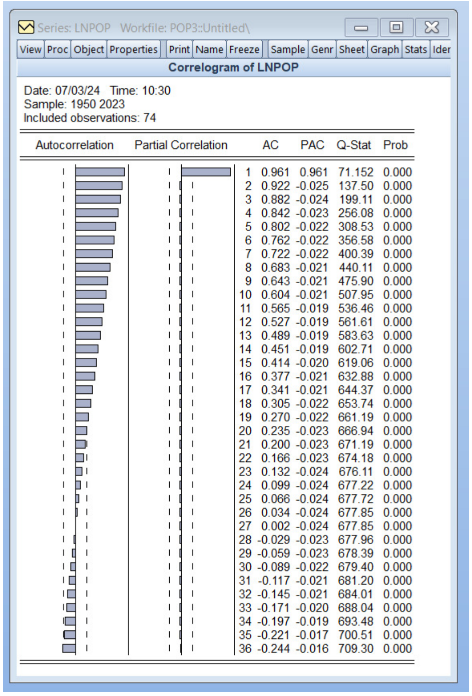
Correlogram of POP_WT

Date: 07/03/24 Time: 10:26
Sample: 1950 2023
Included observations: 74

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	0.959	0.959	70.866	0.000
		2	0.918	-0.025	136.67	0.000
		3	0.876	-0.025	197.50	0.000
		4	0.835	-0.024	253.49	0.000
		5	0.793	-0.023	304.76	0.000
		6	0.752	-0.022	351.47	0.000
		7	0.710	-0.021	393.82	0.000
		8	0.669	-0.020	431.98	0.000
		9	0.629	-0.019	466.17	0.000
		10	0.589	-0.018	496.61	0.000
		11	0.549	-0.016	523.54	0.000
		12	0.511	-0.017	547.18	0.000
		13	0.473	-0.017	567.78	0.000
		14	0.435	-0.018	585.54	0.000
		15	0.398	-0.020	600.67	0.000
		16	0.362	-0.021	613.38	0.000
		17	0.326	-0.021	623.87	0.000
		18	0.291	-0.022	632.36	0.000
		19	0.256	-0.022	639.03	0.000
		20	0.221	-0.022	644.12	0.000
		21	0.187	-0.022	647.83	0.000
		22	0.153	-0.022	650.37	0.000
		23	0.120	-0.022	651.97	0.000
		24	0.088	-0.023	652.84	0.000
		25	0.056	-0.023	653.21	0.000
		26	0.025	-0.023	653.28	0.000
		27	-0.006	-0.023	653.28	0.000
		28	-0.036	-0.023	653.44	0.000
		29	-0.065	-0.022	653.96	0.000
		30	-0.093	-0.022	655.07	0.000
		31	-0.120	-0.021	656.96	0.000
		32	-0.147	-0.021	659.84	0.000

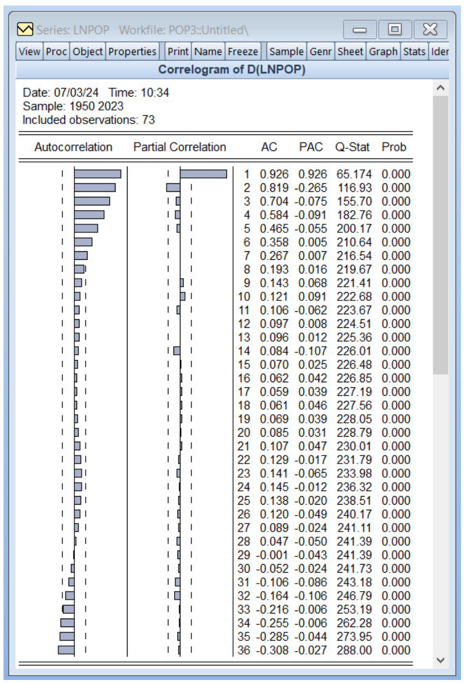
Figure 4. POP's autocorrelation and partial autocorrelation functions.

5.2.1. Estimate the parameters for data and model with stability condition Using the Python program, we calculate the parameters for the proposed non-linear model that are appropriate for the research data under examination. The



	AC	PAC	Q-Stat	Prob
1	0.961	0.961	71.152	0.000
2	0.922	-0.025	137.50	0.000
3	0.882	-0.024	199.11	0.000
4	0.842	-0.023	256.08	0.000
5	0.802	-0.022	308.53	0.000
6	0.762	-0.022	356.58	0.000
7	0.722	-0.022	400.39	0.000
8	0.683	-0.021	440.11	0.000
9	0.643	-0.021	475.90	0.000
10	0.604	-0.021	507.95	0.000
11	0.565	-0.019	536.46	0.000
12	0.527	-0.019	561.61	0.000
13	0.489	-0.019	583.63	0.000
14	0.451	-0.019	602.71	0.000
15	0.414	-0.020	619.06	0.000
16	0.377	-0.021	632.88	0.000
17	0.341	-0.021	644.37	0.000
18	0.305	-0.022	653.74	0.000
19	0.270	-0.022	661.19	0.000
20	0.235	-0.023	666.94	0.000
21	0.200	-0.023	671.19	0.000
22	0.166	-0.023	674.18	0.000
23	0.132	-0.024	676.11	0.000
24	0.099	-0.024	677.22	0.000
25	0.066	-0.024	677.72	0.000
26	0.034	-0.024	677.85	0.000
27	0.002	-0.024	677.85	0.000
28	-0.029	-0.023	677.96	0.000
29	-0.059	-0.023	678.39	0.000
30	-0.089	-0.022	679.40	0.000
31	-0.117	-0.021	681.20	0.000
32	-0.145	-0.021	684.01	0.000
33	-0.171	-0.020	688.04	0.000
34	-0.197	-0.019	693.48	0.000
35	-0.221	-0.017	700.51	0.000
36	-0.244	-0.016	709.30	0.000

Figure 5. LPOP Graphs of autocorrelation and partial autocorrelation functions



	AC	PAC	Q-Stat	Prob
1	0.926	0.926	65.174	0.000
2	0.819	-0.265	116.93	0.000
3	0.704	-0.075	155.70	0.000
4	0.584	-0.091	182.76	0.000
5	0.465	-0.055	200.17	0.000
6	0.358	0.005	210.64	0.000
7	0.267	0.007	216.54	0.000
8	0.193	0.016	219.67	0.000
9	0.143	0.068	221.41	0.000
10	0.121	0.091	222.68	0.000
11	0.106	-0.062	223.67	0.000
12	0.097	0.008	224.51	0.000
13	0.096	0.012	225.36	0.000
14	0.084	-0.107	226.01	0.000
15	0.070	0.025	226.48	0.000
16	0.062	0.042	226.85	0.000
17	0.059	0.039	227.19	0.000
18	0.061	0.046	227.56	0.000
19	0.069	0.039	228.05	0.000
20	0.085	0.031	228.79	0.000
21	0.107	0.047	230.01	0.000
22	0.129	-0.017	231.79	0.000
23	0.141	-0.065	233.98	0.000
24	0.145	-0.012	236.32	0.000
25	0.138	-0.020	238.51	0.000
26	0.120	-0.049	240.17	0.000
27	0.089	-0.024	241.11	0.000
28	0.047	-0.050	241.39	0.000
29	-0.001	-0.043	241.39	0.000
30	-0.052	-0.024	241.73	0.000
31	-0.106	-0.086	243.18	0.000
32	-0.164	-0.106	246.79	0.000
33	-0.216	-0.006	253.19	0.000
34	-0.255	-0.006	262.28	0.000
35	-0.285	-0.044	273.95	0.000
36	-0.308	-0.027	288.00	0.000

Figure 6. Graphs of autocorrelation and partial autocorrelation functions for LDPOP=Zt

program starts by employing different initial values for the estimated real parameters. Then $c = c_1 = 0.99$ and $d = d_1 = 0.001$. By using Equation (2), we get that $w_t = \left[0.99 + 0.001\left(\frac{1}{-(w_{t-1})+1}\right)\right] w_{t-1} + \varepsilon_t$, $w_{t-1} \neq 1$

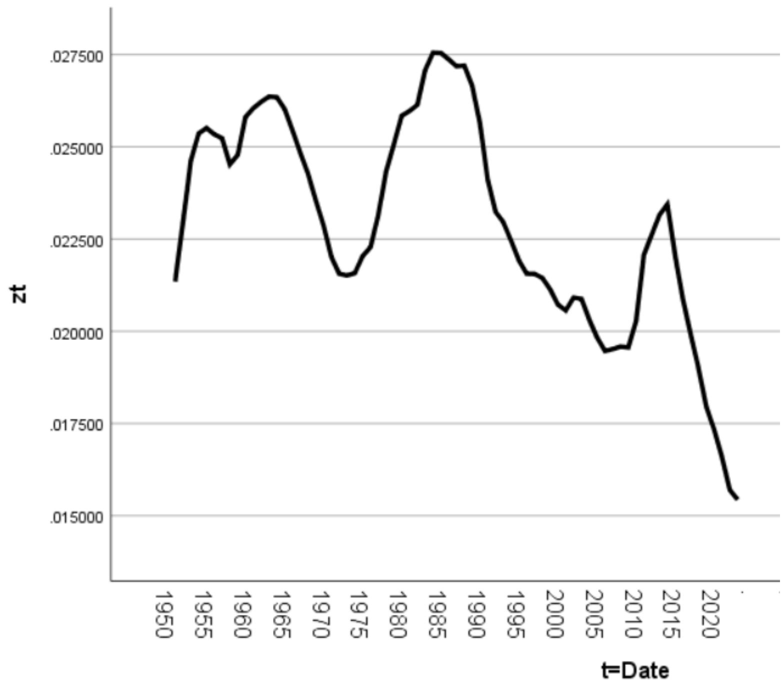


Figure 7. DLPOP= Zt , plot.

5.2.2. The forecasting for study model with transform data We were forecasting by using the proposed model with the estimated parameters that we found in the Python program in the link: https://colab.research.google.com/drive/1ftG-Rwh28y3yGGLHCiBfb90odjeZK_LB#scrollTo=RVpBFxJKzyXS.

We can see Appendix (B) for ttransformingreal data, forecasting ddata,and residuals of anon linear study mmodel,and the statistical criteria were tthosein Table (1).

Table 1. Suggested model of order one

MSE	1.881133792042761e-05
RMSE	0.004337203928849508
MAE	0.0032870004737025407
MAPE	0.018569130213584677
R2	0.9999247474516711
AIC	25.70730362969327
BIC	-794.5621041122428
NBIC	-10.737325731246523

Table 1 clearly illustrates the comparison between the two models. The first-order model meets the stability requirement, whereas the second-order model does not. Additionally, statistical criteria for the second model outperform those of the first model. Consequently, the first-order model was selected for predicting future population observations in Egypt over the next decade and the next figures, figure (8), figure (9) show that the transform original data and forecasted data by using the suggested proposed model.

5.2.3. The residuals test for the suggested model The autocorrelation function and the partial autocorrelation function were found for the residuals of the model proposed in the first-order study for the transferred data,

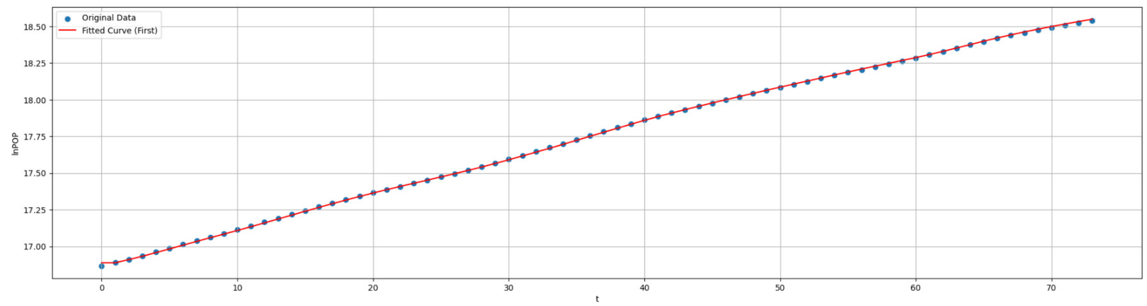


Figure 8. Plot the transformed real data alongside the forecasted data using the suggested model.

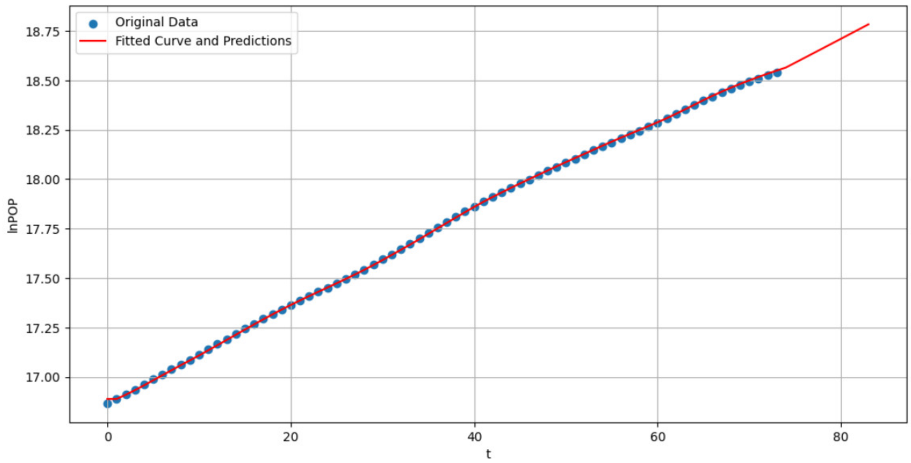


Figure 9. the transformed real data alongside the predicted data using the suggested model.

most of which fall within the limits of confidence, which indicates that the model agrees with the data used and the possibility of using it to predict future values. The next figures, Figure (10) and Figure (11), explained the autocorrelation and partial autocorrelation functions graphs for the study model residuals. The residual is clearly white noise, indicating that the model is correct.

5.2.4. Forecasting POP data for the next ten years that we used by using suggested model We can see the program in the Python in the link: https://colab.research.google.com/drive/1dlTXKjH3l5pmFGBW1AfWNxJp0KeGvzMI#scrollTo=CoDEeEh2pmem.

The Python program that found the forecasting POP data by using the study model with the original data. The next figure, Figure (12), shows the forecasting of POP data by using the suggested model.

We can see Appendix (C) for the real data POP, forecasting data and residuals of a nonlinearstudy model.

5.2.5. The prediction for suggested model with real data We can see the prediction for the study model with real data by using python program in the link https://colab.research.google.com/drive/1dlTXKjH3l5pmFGBW1AfWNxJp0KeGvzMI#scrollTo=CoDEeEh2pmem. for the next ten years. The next figure, figure (13), explain the prediction of the real data POP and the real data that we used in our search by using the study model for the next ten years.

The next table, Table ((2),shows the predictions for POP data by using the proposed study model for the next ten years.

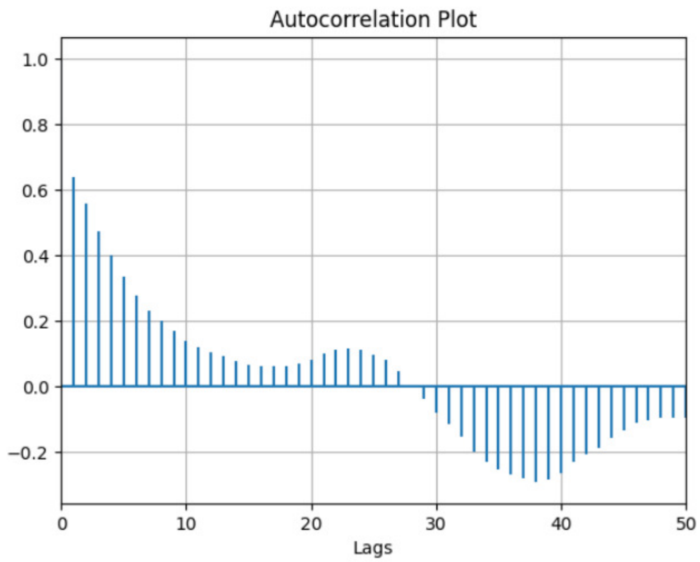


Figure 10. The aautocorrelation function for the suggested model residuals

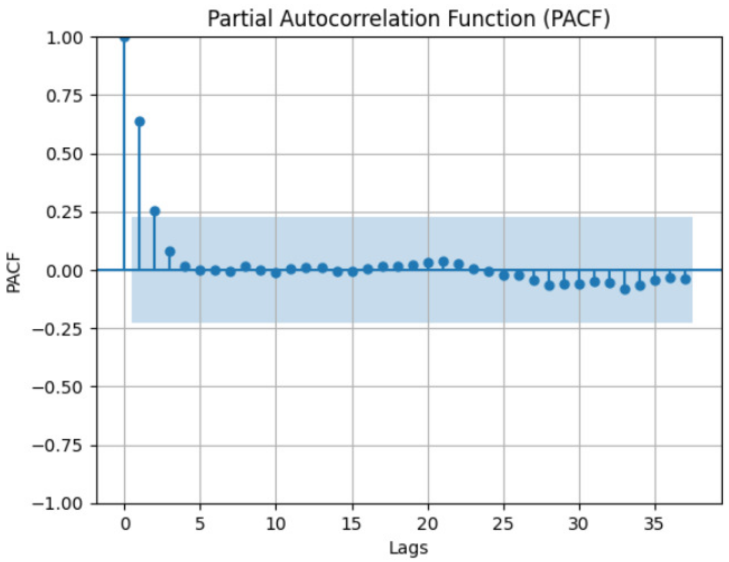


Figure 11. The partial aautocorrelationfunction for the suggested model residuals.

6. Conclusions

The paper presents a valuable and innovative approach for forecasting the population of the Arab Republic of Egypt using a nonlinear autoregressive model. The proposed model demonstrates superior performance in capturing the underlying patterns of population dynamics and provides a reliable tool for decision makers. However, several weaknesses need to be addressed to enhance the robustness and practical applicability of the findings. By incorporating the recommended improvements, the study could provide a more comprehensive and actionable framework for population forecasting.

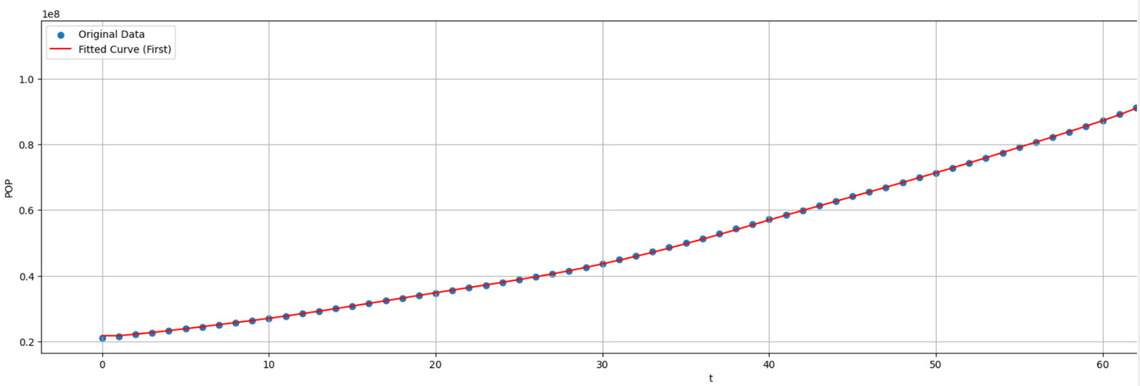


Figure 12. Forecasting POP data by using the suggested model

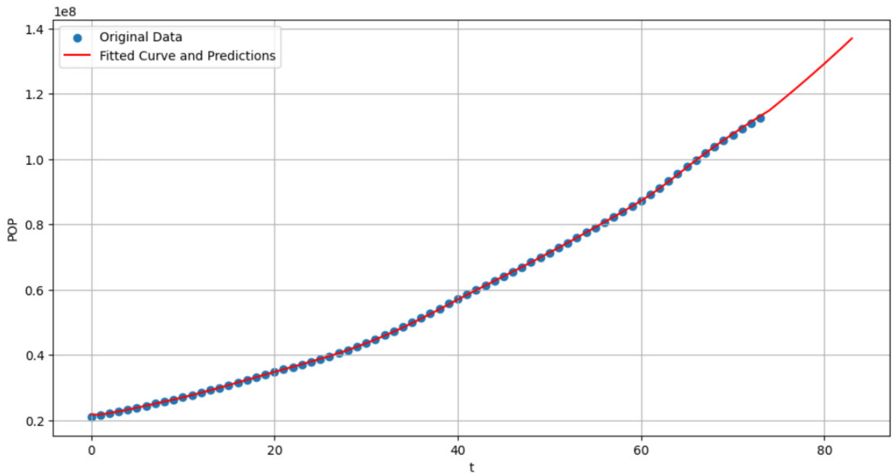


Figure 13. predictions for POP data by using the study model for the next ten years.

Table 2. Prediction of POP by using study model

Years	Prediction of Egypt POP	Population Growth Rate
2024	114,956,063	1.55%
2025	117,235,413	1.98%
2026	119,555,356	1.97%
2027	121,916,617	1.97%
2028	124,319,931	1.97%
2029	126,766,046	1.96%
2030	129,255,726	1.96%
2031	131,789,746	1.96%
2032	134,368,896	1.95%
2033	136,993,980	1.95%

7. Suggestions for the future:

1. The proposed model has potential for use in future research involving diverse real datasets to predict future observations of the real phenomena under study.
2. Researchers can define stability conditions, including single-point stability conditions for the proposed model ranging from second-order to p-order.
3. Models similar to those presented in this research can be proposed and used. The stability conditions can then be derived and applied to numerical examples. These models can also be effectively used to predict future observations of real data in subsequent research in a manner similar to this one.

You can refer to the following appendices:

1. Appendix (A) regarding the population of Egypt from 1950 to 2023.
2. Appendix (B) for predicting the converted data using the proposed model.
3. Appendix (C) for forecasting the original data using the proposed model.

Appendix (A): The population series data between 1950 to 2023 in Egypt.

t	Pop = Wt	t	Pop = Wt	t	Pop = Wt	t	Pop = Wt
1950	21150058	1971	35555969	1992	59,989,142	2013	93,377,890
1951	21606363	1972	36,330,767	1993	61,382,200	2014	95,592,324
1952	22108163	1973	37,120,775	1994	62,775,847	2015	97,723,798
1953	22659254	1974	37,930,374	1995	64,166,907	2016	99,784,029
1954	23241217	1975	38,775,583	1996	65,565,194	2017	101,789,385
1955	23841701	1976	39,649,050	1997	66,993,728	2018	103,740,765
1956	24453497	1977	40,577,356	1998	68,446,011	2019	105,618,671
1957	25078378	1978	41,576,636	1999	69,907,886	2020	107,465,134
1958	25700968	1979	42,632,458	2000	71,371,371	2021	109,262,177
1959	26345760	1980	43,748,555	2001	72,854,260	2022	110,990,103
1960	27034498	1981	44,899,572	2002	74,393,759	2023	112,716,598
1961	27747866	1982	46,088,647	2003	75,963,322	–	–
1962	28485022	1983	47,353,665	2004	77,522,426	–	–
1963	29245936	1984	48,676,442	2005	79,075,309	–	–
1964	30026648	1985	50,035,843	2006	80,629,669	–	–
1965	30818469	1986	51,424,313	2007	82,218,755	–	–
1966	31613132	1987	52,841,318	2008	83,844,782	–	–
1967	32408414	1988	54,298,445	2009	85,501,063	–	–
1968	33204629	1989	55,765,843	2010	87,252,413	–	–
1969	33995955	1990	57,214,630	2011	89,200,053	–	–
1970	21150058	1991	58,611,032	2012	91,240,376	–	–

Appendix (B): Log(Pop) values, forecasted values, and residuals using the non-linear study model

t	logPop	Predict logPop	Residual	t	logPop	Predict logpop	Residual
1950	16.86715321	16.88855	-0.02139697	1992	17.90967414	17.9092	0.00046917
1951	16.88849841	16.88855	-5.1766E-05	1993	17.93263045	17.93248	0.0001535
1952	16.91145746	16.90992	0.0015333	1994	17.95508096	17.95546	-0.00038326
1953	16.93607889	16.93291	0.00316472	1995	17.97699817	17.97795	-0.00094683
1954	16.96143786	16.95757	0.00386906	1996	17.99855553	17.99989	-0.00133624
1955	16.98694675	16.98296	0.00398478	1997	18.02010956	18.02148	-0.00136864
1956	17.01228379	17.00851	0.00377853	1998	18.04155583	18.04306	-0.00150547
1957	17.0375166	17.03388	0.00364013	1999	18.06268902	18.06454	-0.00184747
1958	17.06203921	17.05914	0.00289591	2000	18.08340738	18.0857	-0.00229079
1959	17.08681791	17.0837	0.00311893	2001	18.10397157	18.10644	-0.0024729
1960	17.11262431	17.10851	0.00411321	2002	18.12488261	18.12704	-0.00215378
1961	17.13866949	17.13435	0.00431719	2003	18.14576118	18.14798	-0.00221445
1962	17.16488896	17.16043	0.00445636	2004	18.16607782	18.16888	-0.00280454
1963	17.19125118	17.18669	0.00456376	2005	18.18591123	18.18923	-0.00331516
1964	17.21759581	17.21309	0.00451062	2006	18.20537724	18.20909	-0.00370931
1965	17.24362471	17.23947	0.00415936	2007	18.224894	18.22858	-0.00368481
1966	17.26908316	17.26553	0.00355381	2008	18.24447781	18.24812	-0.00364408
1967	17.29392864	17.29102	0.00290651	2009	18.26403937	18.26773	-0.00369274
1968	17.31819985	17.3159	0.00229873	2010	18.28431577	18.28732	-0.00300428
1969	17.3417521	17.34021	0.00154704	2011	18.30639219	18.30762	-0.0012316
1970	17.36461014	17.36379	0.00082107	2012	18.32900808	18.32973	-0.0007219
1971	17.3866186	17.38668	-5.933E-05	2013	18.35216515	18.35238	-0.00021122
1972	17.40817552	17.40872	-0.00054055	2014	18.37560308	18.37556	3.8415E-05
1973	17.42968734	17.4303	-0.00061472	2015	18.39765567	18.39903	-0.00137853
1974	17.45126277	17.45184	-0.00058012	2016	18.4185187	18.42112	-0.00259783
1975	17.4733013	17.47345	-0.00014611	2017	18.43841638	18.44201	-0.00359131
1976	17.49557755	17.49552	6.1889E-05	2018	18.4574057	18.46193	-0.00452651
1977	17.51872073	17.51782	0.00089878	2019	18.47534572	18.48095	-0.00560141
1978	17.54304893	17.541	0.00205259	2020	18.49267702	18.49891	-0.00623432
1979	17.56812645	17.56536	0.0027691	2021	18.50926085	18.51627	-0.00700517
1980	17.59396914	17.59047	0.00350046	2022	18.52495159	18.53287	-0.00792062
1981	17.61993882	17.61635	0.0035926	2023	18.54038724	18.54858	-0.00819687
1982	17.64607721	17.64235	0.00372629	2024	–	18.56404	–
1983	17.67315478	17.66852	0.00463022	2025	–	18.58773	–
1984	17.70070573	17.69564	0.00506709	2026	–	18.61144	–
1985	17.72825017	17.72323	0.00502342	2027	–	18.63519	–
1986	17.75562163	17.75081	0.0048133	2028	–	18.65897	–
1987	17.78280398	17.77822	0.00458728	2029	–	18.68279	–
1988	17.81000615	17.80544	0.00457044	2030	–	18.70663	–
1989	17.83667211	17.83267	0.00399755	2031	–	18.73051	–
1990	17.86232019	17.85938	0.00294371	2032	–	18.75442	–
1991	17.8864335	17.88506	0.00137435	2033	–	18.77836	–

Appendix (c): Pop values, forecasting pop values, and residuals using the non-linear study model.

t	Pop.	Predict Pop.	Residual	t	Pop	Predict Pop.	Residual
1950	21150058	21758763	-608705	1992	59,989,142	59886901	102241
1951	21606363	21758763	-152400	1993	61,382,200	61289554	92646
1952	22108163	22223195	-115032	1994	62,775,847	62707422	68425
1953	22659254	22733932	-74678	1995	64,166,907	64125889	41018
1954	23241217	23294837	-53620	1996	65,565,194	65541724	23470
1955	23841701	23887165	-45464	1997	66,993,728	66964913	28815
1956	24453497	24498343	-44846	1998	68,446,011	68418889	27122
1957	25078378	25121035	-42657	1999	69,907,886	69897037	10849
1958	25700968	25757045	-56077	2000	71,371,371	71384947	-13576
1959	26345760	26390723	-44963	2001	72,854,260	72874496	-20236
1960	27034498	27046998	-12500	2002	74,393,759	74383795	9964
1961	27747866	27748003	-137	2003	75,963,322	75950711	12611
1962	28485022	28474075	10947	2004	77,522,426	77548228	-25802
1963	29245936	29224360	21576	2005	79,075,309	79135099	-59790
1964	30026648	29998825	27823	2006	80,629,669	80715638	-85969
1965	30818469	30793441	25028	2007	82,218,755	82297680	-78925
1966	31613132	31599364	13768	2008	83,844,782	83915067	-70285
1967	32408414	32408180	234	2009	85,501,063	85,570053	-68990
1968	33204629	33217626	-12997	2010	87,252,413	87255832	-3419
1969	33995955	34028021	-32066	2011	89,200,053	89038372	161681
1970	34781985	34833440	-51455	2012	91,240,376	91020699	219677
1971	35555969	35633469	-77500	2013	93,377,890	93097359	280531
1972	36,330,767	36421237	-90470	2014	95,592,324	95272941	319383
1973	37,120,775	37209834	-89059	2015	97,723,798	97526814	196984
1974	37,930,374	38013912	-83538	2016	99,784,029	99696248	87781
1975	38,775,583	38837929	-62346	2017	101,789,385	101793171	-3786
1976	39,649,050	39698191	-49141	2018	103,740,765	103834241	-93476
1977	40,577,356	40587214	-9858	2019	105,618,671	105820375	-201704
1978	41,576,636	41532053	44583	2020	107,465,134	107731725	-266591
1979	42,632,458	42549130	83328	2021	109,262,177	109611073	-348896
1980	43,748,555	43623755	124800	2022	110,990,103	111440121	-450018
1981	44,899,572	44759730	139842	2023	112,716,598	113198820	-482222
1982	46,088,647	45931246	157401	–	–	–	–
1983	47,353,665	47141498	212167	–	–	–	–
1984	48,676,442	48429045	247397	–	–	–	–
1985	50,035,843	49775380	260463	–	–	–	–
1986	51,424,313	51158991	265322	–	–	–	–
1987	52,841,318	52572189	269129	–	–	–	–
1988	54,298,445	54014431	284014	–	–	–	–
1989	55,765,843	55497509	268334	–	–	–	–
1990	57,214,630	56991040	223590	–	–	–	–
1991	58,611,032	58465630	145402	–	–	–	–

REFERENCES

1. T. Ozaki, *The statistical analysis of perturbed limit cycle processes using nonlinear time series models*, Journal of Time Series Analysis, vol. 3, no. 1, pp. 29–41, 1982.
2. T. Ozaki, *Nonlinear time series models and dynamical system*, in Nonlinear Time Series Models and Dynamical System, vol. 5, Elsevier Science Publishers B.V., 1985, pp. 25–83.
3. A. I. Saba and A. H. Elsheikh, *Forecasting the prevalence of COVID-19 outbreak in Egypt using nonlinear autoregressive artificial neural networks*, Process Safety and Environmental Protection, vol. 141, pp. 1–8, 2020.
4. B. G. Al-Ani, *Statistical modeling of the novel COVID-19 epidemic in Iraq*, Epidemiological Method, vol. 10, no. s1, pp. 1–16, 2021.
5. Q. M. Abdulqader, *Forecasting the Ratio of the Rural Population in Iraq Using Box-Jenkins Methodology*, Scientific Journal of University of Zakho, vol. 11, no. 1, pp. 132–136, 2023.
6. M. R. Abonazel and N. M. Darwish, *Forecasting confirmed and recovered COVID-19 cases and deaths in Egypt after the genetic mutation of the virus: ARIMA Box-Jenkins approach*, 2022, pp. 1–23.
7. M. A. Zabadi, R. Assaf, and M. Kanan, *A mathematical and statistical approach for predicting the population growth*, International Journal, vol. 3, no. 7, pp. 50–59, 2017.
8. R. Baragona, F. Battaglia, and D. Cucina, *A note on estimating autoregressive exponential models*, Quaderni di Statistica, vol. 4, pp. 1–18, 2002.
9. A. Ahmed, B. Salam, M. Mohammad, A. Akgül, and S. H. A. Khoshnaw, *Analysis coronavirus disease (COVID-19) model using numerical approaches and logistic model*, AIMS Bioengineering, vol. 7, no. 3, pp. 130–146, 2020.
10. A. A. Khadom, A. K. Al-Jiboory, M. S. Mahdi, and H. B. Mahood, *Regression and validation studies of the spread of novel COVID-19 in Iraq using mathematical and dynamic neural networks models*, Caspian Journal of Environmental Sciences, vol. 19, no. 3, pp. 431–440, 2021.
11. C. Mahanty, R. Kumar, B. K. Mishra, D. J. Hemanth, D. Gupta, and A. Khanna, *Prediction of COVID-19 active cases using exponential and non-linear growth models*, Expert Systems, vol. 39, no. 3, pp. 1–22, 2022.
12. R. A. Abdulkadir, S. I. A. Ali, S. I. Abba, and P. Esmaili, *Forecasting of daily rainfall at Ercan Airport Northern Cyprus: A comparison of linear and non-linear models*, Desalination and Water Treatment, vol. 177, pp. 297–305, 2020.
13. M. Ma, M. Su, S. Li, F. Jiang, and R. Li, *Predicting coal consumption in South Africa based on linear, nonlinear, and combined models*, Sustainability, vol. 10, no. 7, 2018.
14. A. S. Youns and S. M. Ahmad, *Stability Conditions for a Nonlinear Time Series Model*, International Journal of Mathematics and Mathematical Sciences, vol. 2023, 2023.
15. S. M. Ahmad, A. S. Youns, and M. S. Hamdi, *Nonlinear Autoregressive Model for Stability and Prediction*, vol. 2025, 2025.
16. M. Y. Al-Ridha, A. S. Anaz, and R. R. O. Al-Nima, *Expecting confirmed and death cases of COVID-19 in Iraq by utilizing backpropagation neural network*, Bulletin of Electrical Engineering and Informatics, vol. 10, no. 4, pp. 2137–2143, 2021.
17. O. B. Shukur, S. H. Ali, and L. A. Saber, *Climatic temperature data forecasting in Nineveh Governorate using the recurrent neural network method*, International Journal of Advanced Science Engineering and Information Technology, vol. 11, no. 1, pp. 113–123, 2021.
18. O. B. Shukur and M. H. Lee, *Daily wind speed forecasting through hybrid KF-ANN model based on ARIMA*, Renewable Energy, vol. 76, pp. 637–647, 2015.
19. L. Olatomiwa, S. Mekhilef, S. Shamshirband, K. Mohammadi, D. Petković, and C. Sudheer, *A support vector machine-firefly algorithm-based model for global solar radiation prediction*, Solar Energy, vol. 115, pp. 632–644, 2015.
20. J. Y. Lee, T. T. Nguyen, H. G. Nguyen, and J. Y. Lee, *Towards Predictive Crude Oil Purchase: A Case Study in the USA and Europe*, Energies, vol. 15, no. 11, 2022.
21. M. S. Bakay and Ü. Ağbulut, *Electricity production based forecasting of greenhouse gas emissions in Turkey with deep learning, support vector machine and artificial neural network algorithms*, Journal of Cleaner Production, vol. 285, p. 125324, 2021.
22. A. E. Gürel, Ü. Ağbulut, and Y. Biçen, *Assessment of machine learning, time series, response surface methodology and empirical models in prediction of global solar radiation*, Journal of Cleaner Production, vol. 277, p. 122353, 2020.
23. T. A. Whittaker and J. E. Miller, *Exploring the Enumeration Accuracy of Cross-Validation Indices in Latent Class Analysis*, Structural Equation Modeling, vol. 28, no. 3, pp. 376–390, 2021.
24. D. M. Wolcott, A. Duarte, and F. W. Weckerly, *Statistical Inference*, Encyclopedia of Ecology, 2nd ed., vol. 2, pp. 199–205, 2019.